Emergence of structure in growing networks

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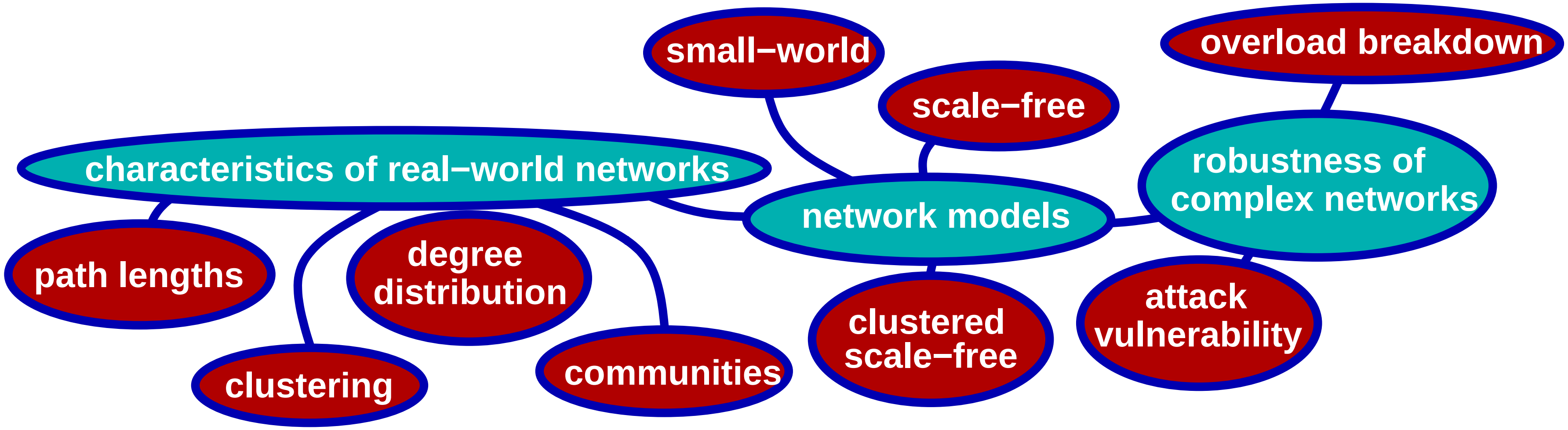
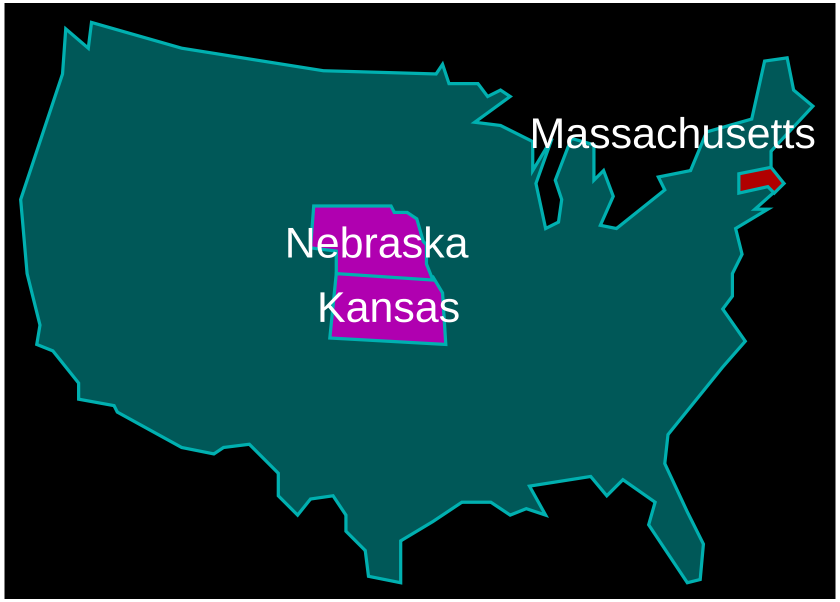




The experiment:

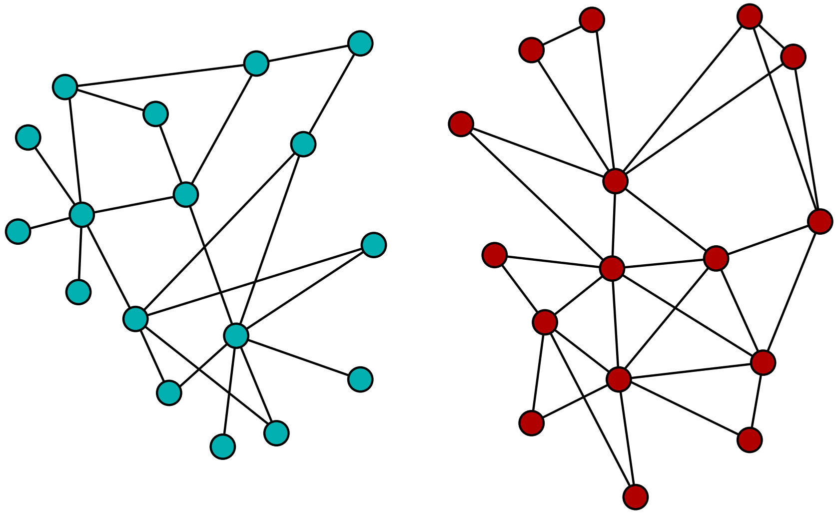
- How many intermediates between people in Nebraska/Kansas and Boston.
- Target known by name and profession.
- Packages sent closer in geographical and social space. To persons at least known by their first name

The result:

- A median of 5 intermediates.
- Comparing with similar experiments on smaller populations $\Rightarrow$ a logarithmically increasing shortest path length.
- Average shortest pathlength of the Earth's population = 6.

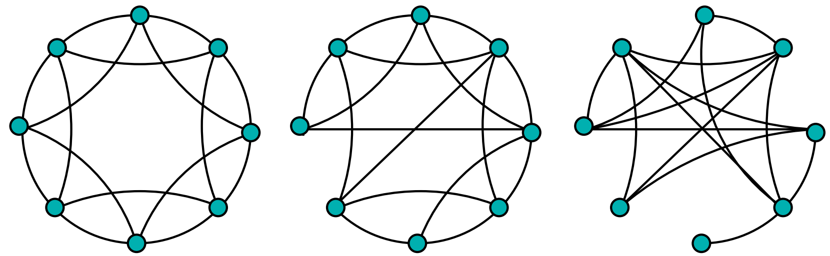
S. Milgram, "The Small-World Problem" *Psychology Today* 2:60-67.

Clustering—network transitivity.



The clustering coefficient:

$$C = \frac{3 \times (\text{\# triangles on the graph})}{(\text{\# connected triples of vertices})}$$

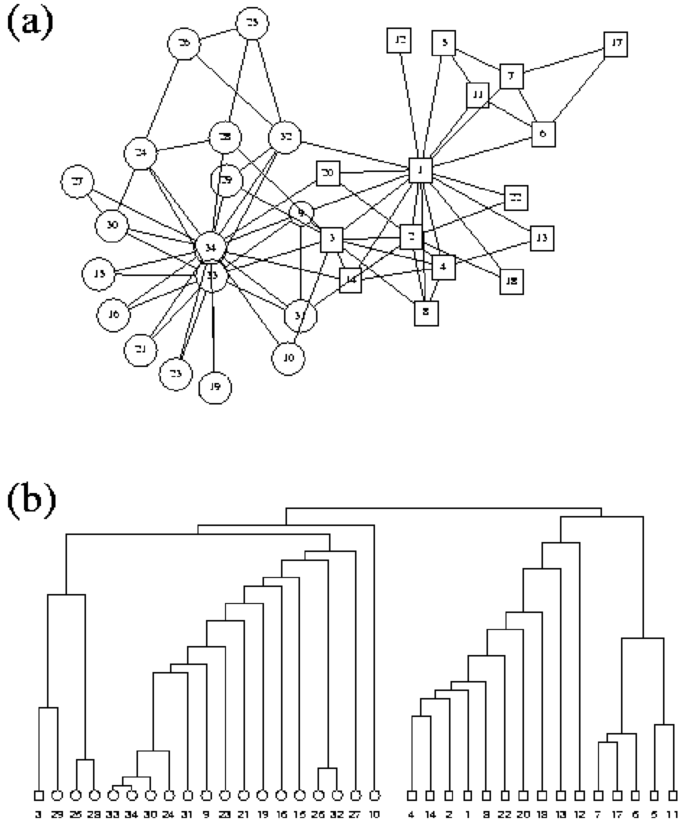


The Watts-Strogatz Model:

- Logarithmically increasing average shortest path length at any finite fraction of rewirings.
- Algebraically decreasing clustering.

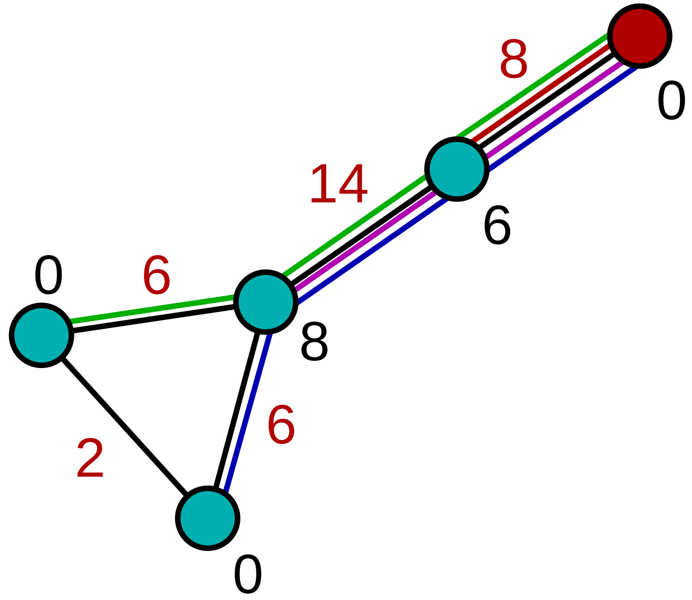
D. J. Watts and S. H. Strogatz, "Collective dynamics of 'small-world' networks" *Nature* 393:440-42.

Community Structure



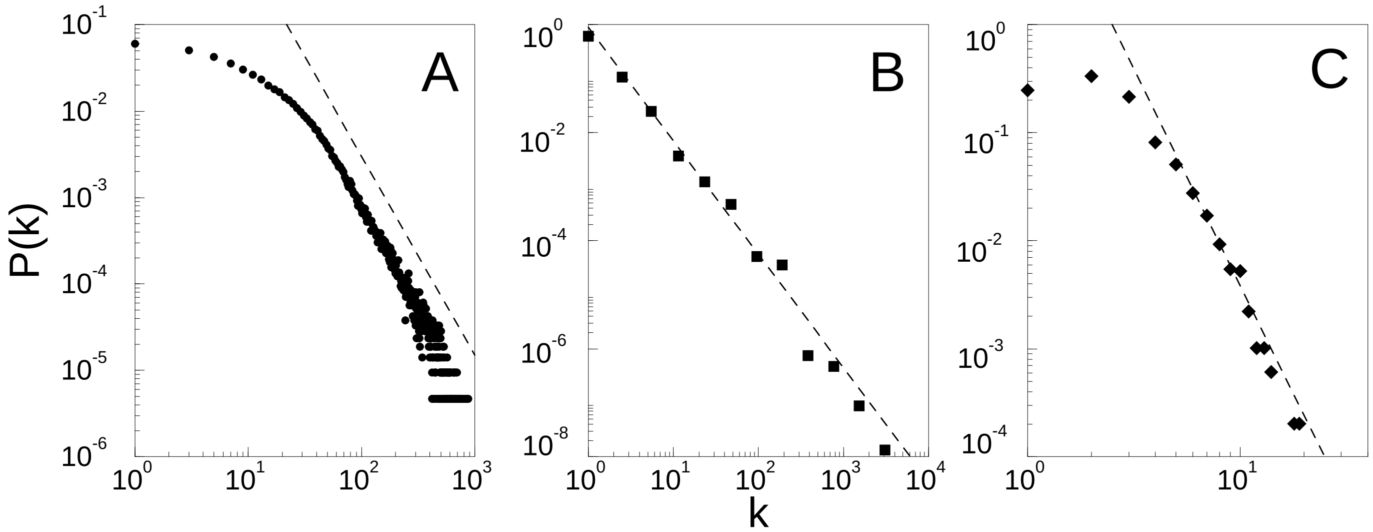
M. Girvan and M. E. J. Newman, "Community structure in social and biological networks" e-print cond-mat/0112110.

Betweenness centrality



$$C_B(v) = \sum_{w \in V \setminus \{u\}} \sum_{u \in V} \frac{\sigma_{uw}(v)}{\sigma_{uw}}$$

$$C_B(e) = \sum_{w \in V \setminus \{v\}} \sum_{v \in V} \frac{\sigma_{vw}(e)}{\sigma_{vw}}$$



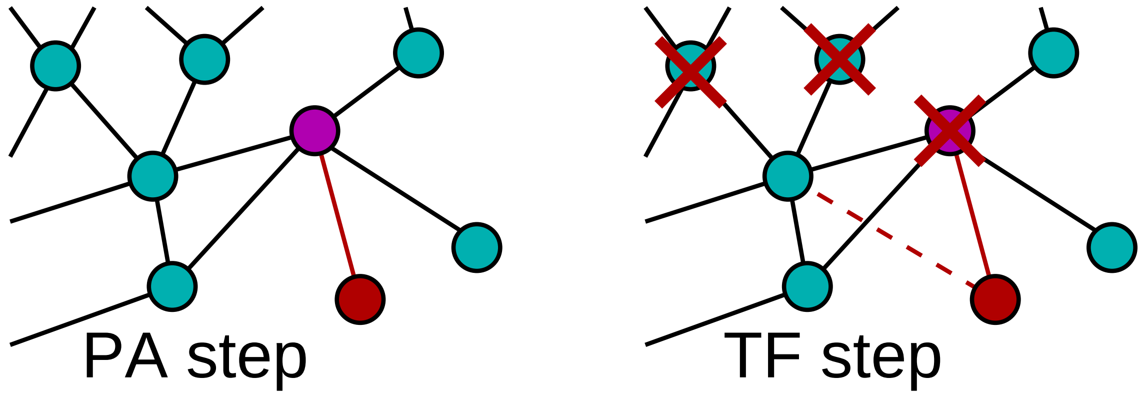
Power-law degree distribution: **A.** Actor collaboration. **B.** WWW. **C.** Power grid.

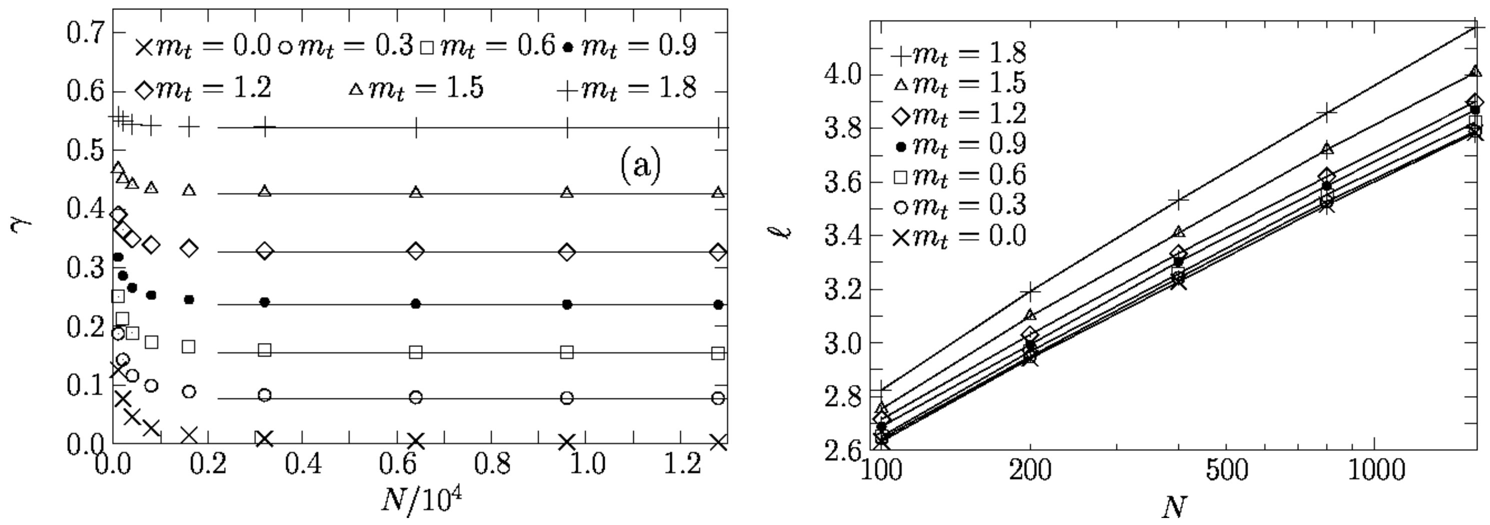
The Barabási-Albert model: **1.** Start with m_0 vertices. **2.** Add one vertex and m edges in each timestep. **3.** Add an edge to a vertex v with a probability proportional to k_v. (Preferential attachment.)

The degree distribution of this model $\propto k^{-3}$.

A.-L. Barabási and R. Albert, *Emergence of Scaling in Random Networks*, Science **286**, 509 (1999).

Scale-free networks with clustering

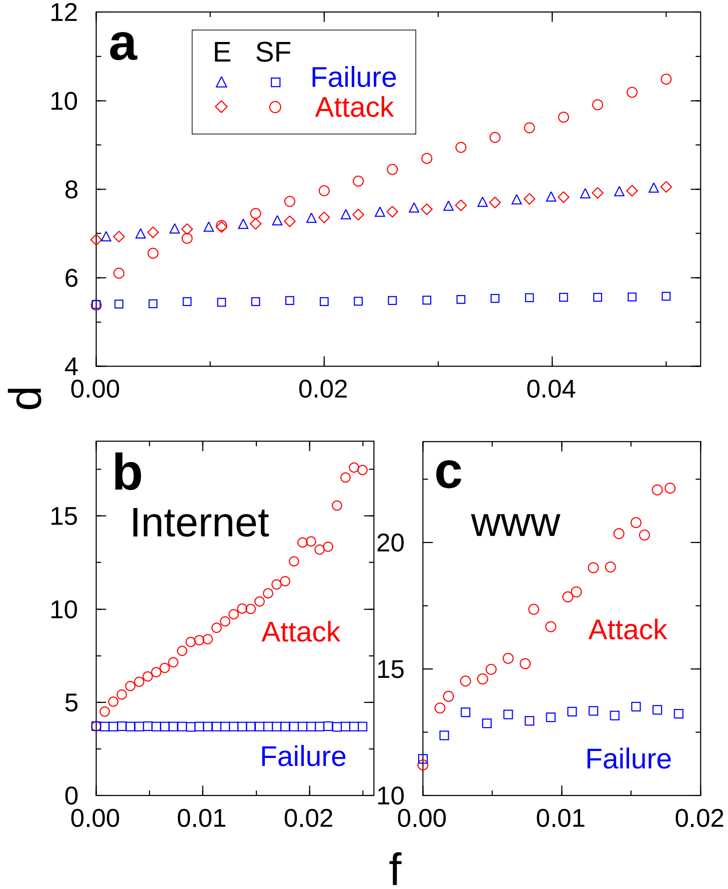


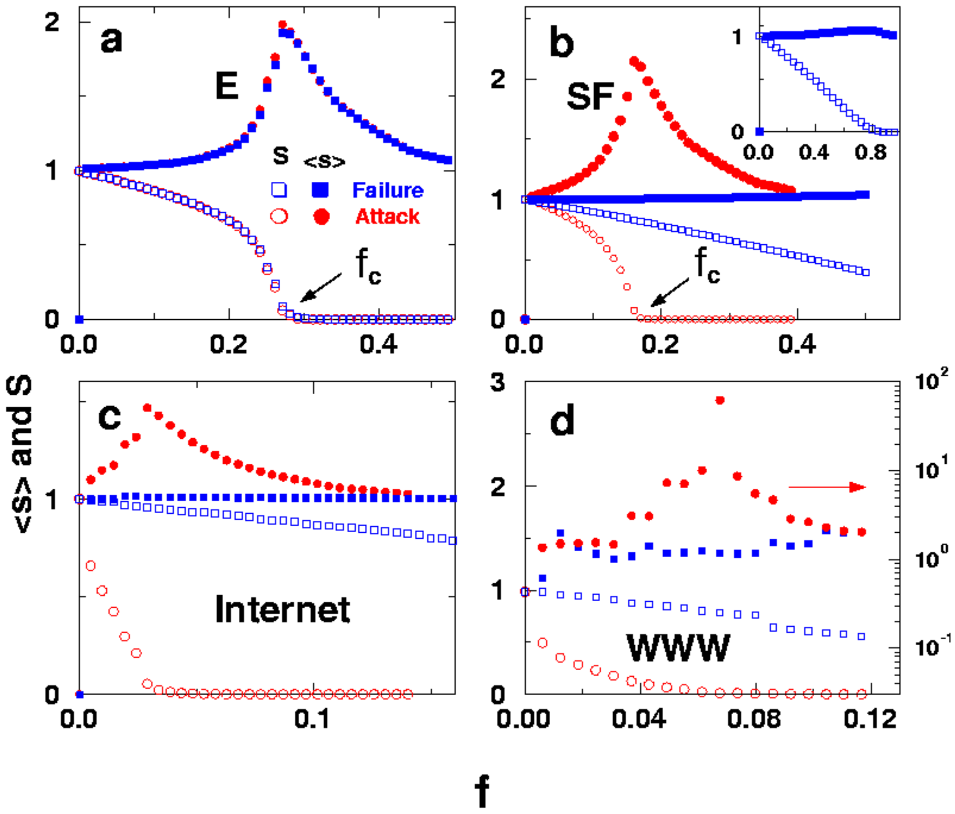


Petter Holme and Beom Jun Kim, *Growing scale-free networks with tunable clustering* Phys. Rev. E **65**, 026107 (2002).

Disadvantages with this kind of modeling

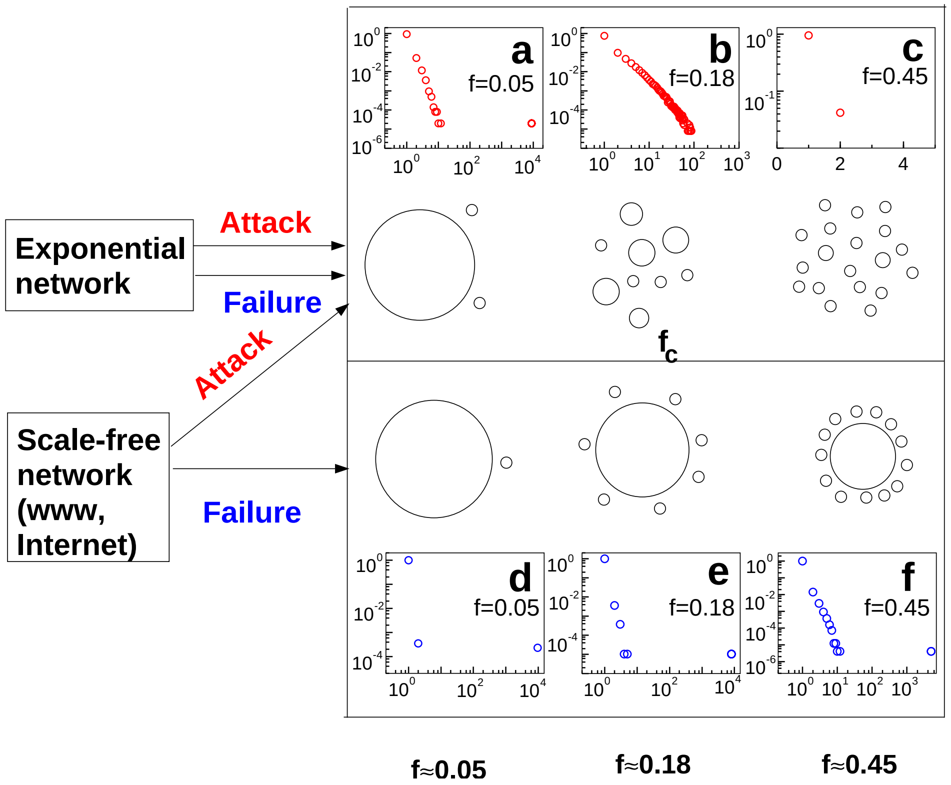
- Doesn't uniformly sample the ensemble of graphs with given values of statistical quantities (such as degree distribution or clustering).
- Conclusions from models must be cautiously extended to real systems.
- The principles motivating the models varies a lot (connection to the real-world may sometimes be completely lacking).





R. Albert, H. Jeong, and A.-L. Barabasi, *Error and Attack Tolerance of Complex Networks*, Nature **406**, 378 (2000).

Attack vulnerability and error tolerance *(continued)*



Attack vulnerability and error tolerance

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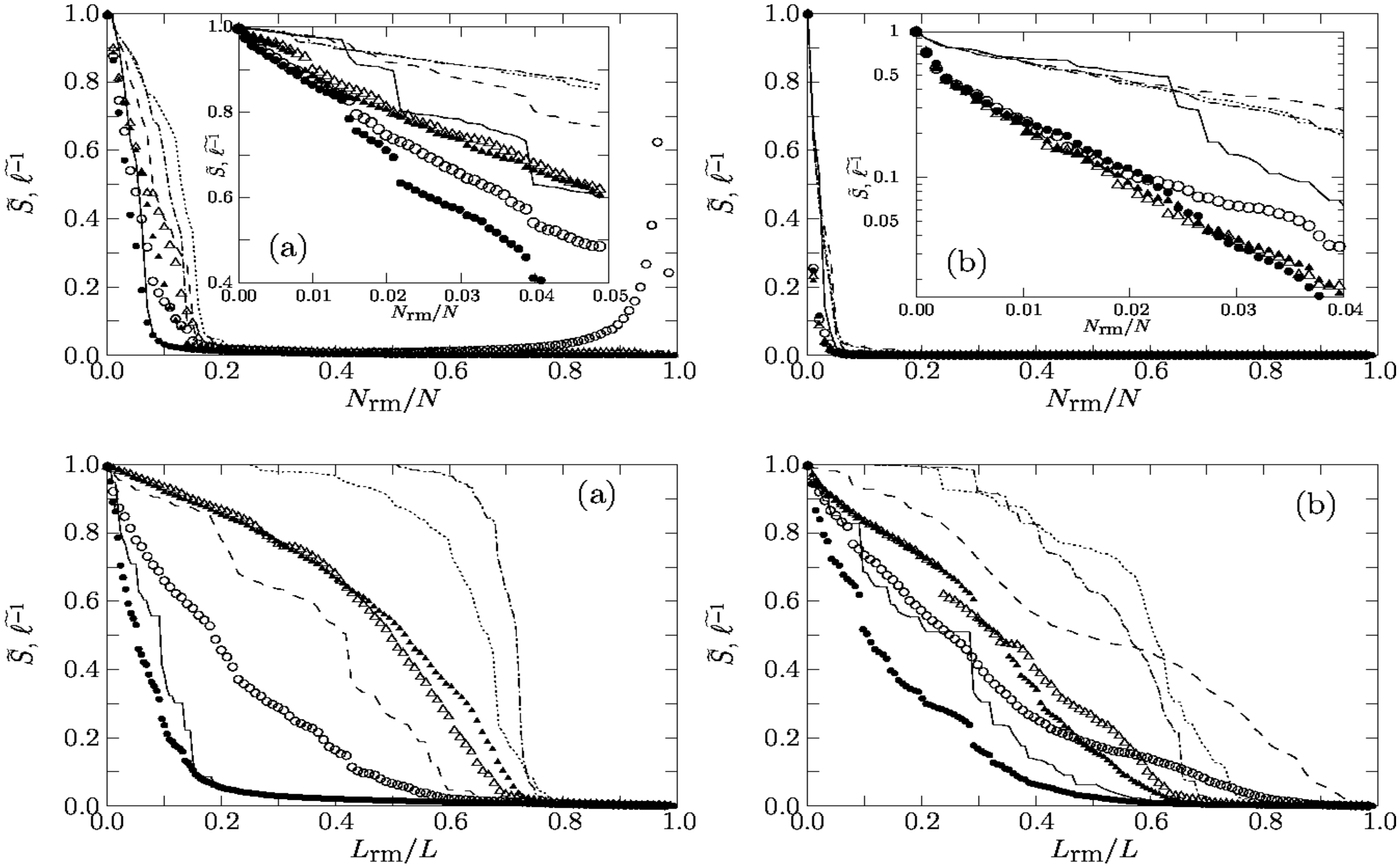
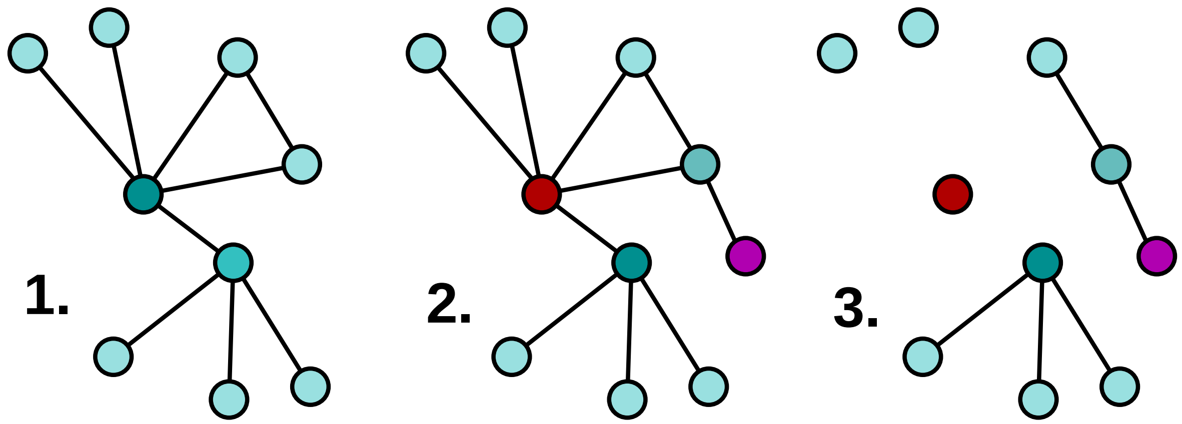
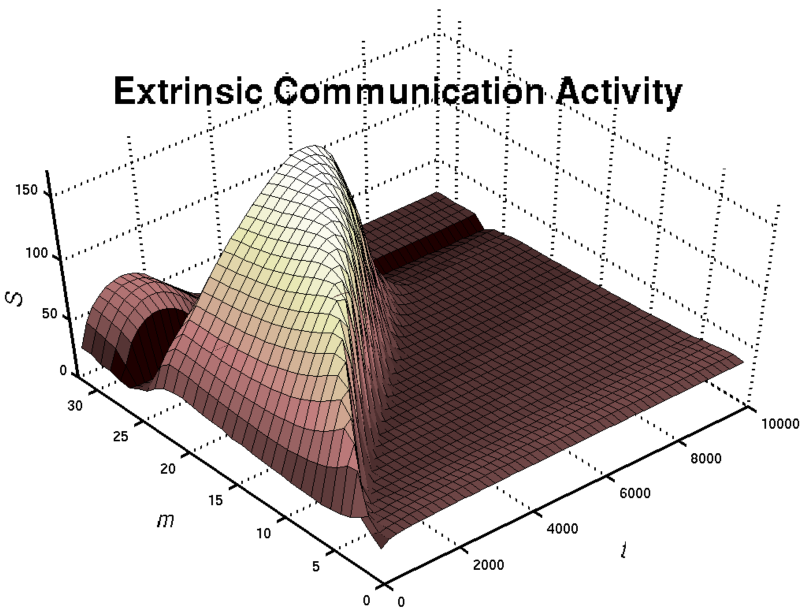
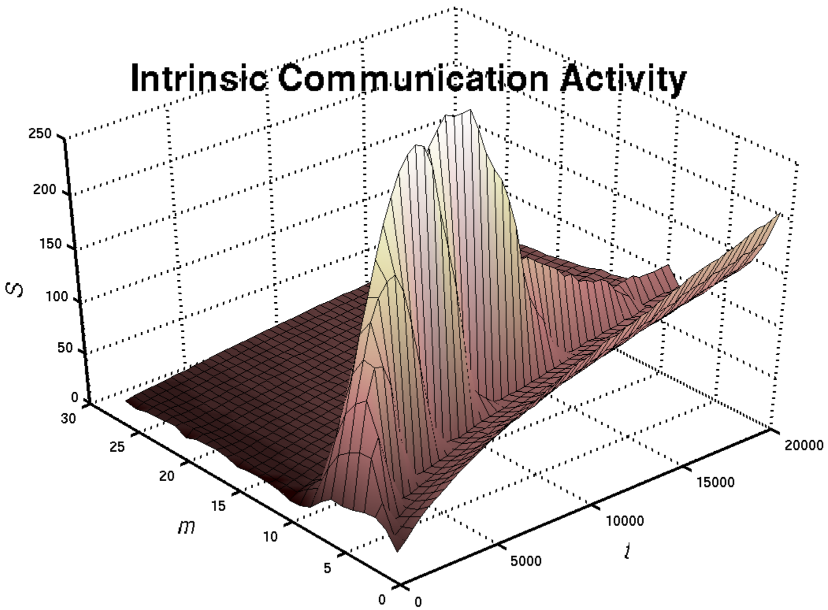


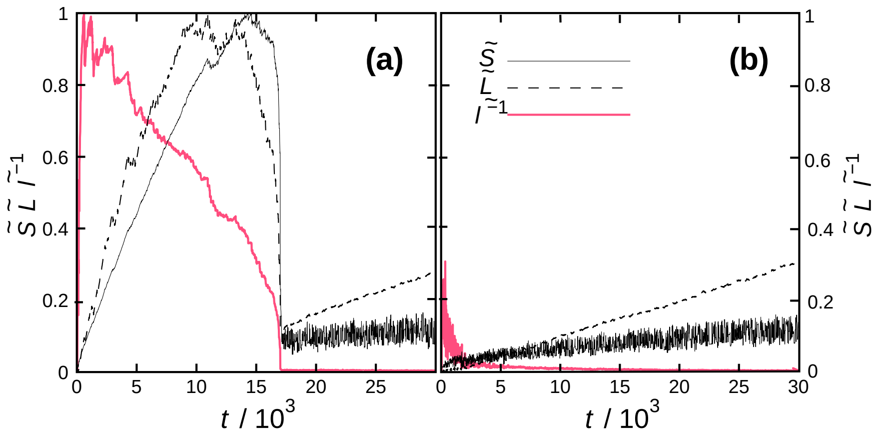
P. Holme, B. J. Kim, C. N. Yoon, and S. K. Han, *Attack vulnerability of complex networks*, to appear in Phys. Rev. E, e-print cond-mat/0202410.

VERTEX OVERLOAD BREAKDOWN









Intrinsic Communication Activity

Petter Holme and Beom Jun Kim, submitted to Phys. Rev. E.