Sphinx’ Helsinki Workshop 2002:

A Zero-Temperature Study of Vortex Mobility in Two-Dimensional Vortex Glass Models

Petter Holme

Department of Theoretical, Physics, Umeå University, Sweden

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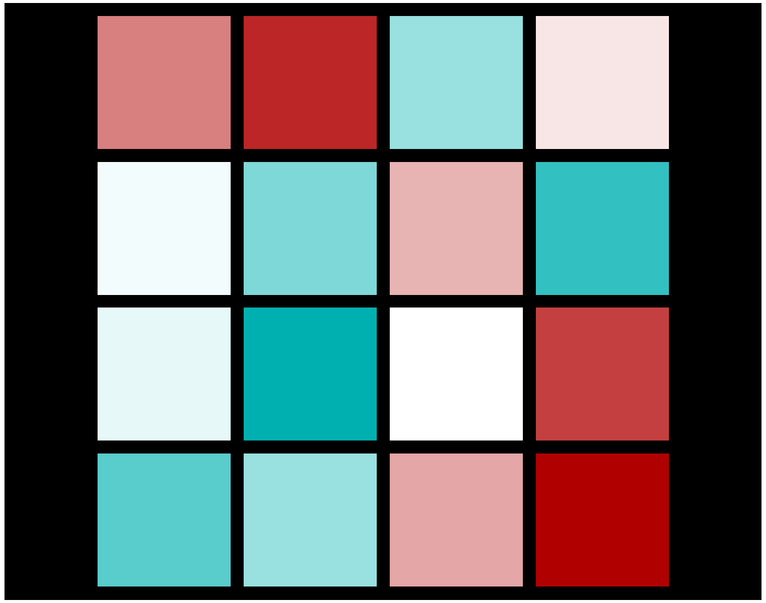
$$\mathcal{H} = -\sum_{(ij)_{\rm nn}} \cos\left(\theta_i - \theta_j - A_{ij} - \frac{\mathbf{r}_{ij}}{L}\cdot\Delta\right)$$

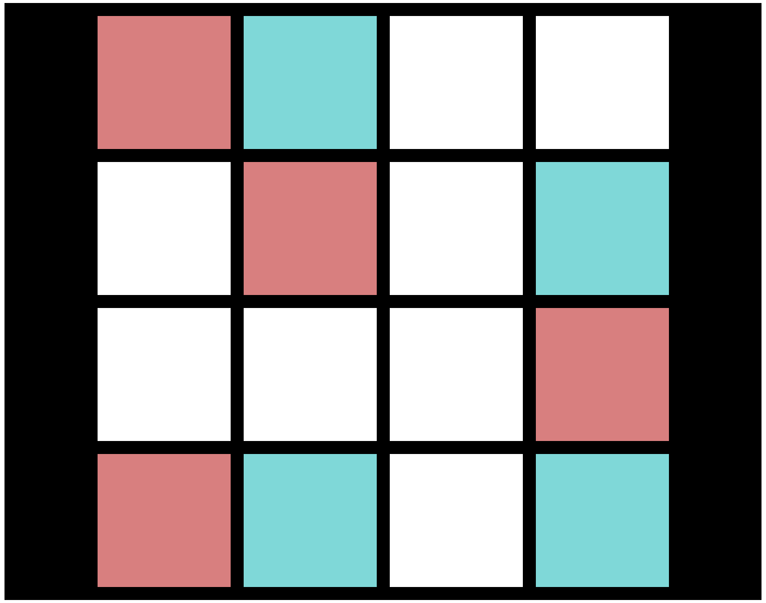
Random Gauge *XY* Model

$A_{ij} \in [-r\pi, r\pi)$, $0 \leqslant r \leqslant 1$. Standard *XY* gauge glass corresponds to $r = 1$.

XY Spin Glass Model

$A_{ij} \in \{0, \pi\}$, $A_{ij} = \pi$ with probability s, $0 \leqslant s \leqslant 1$. Standard *XY* spin glass corresponds to $s = 1/2$.





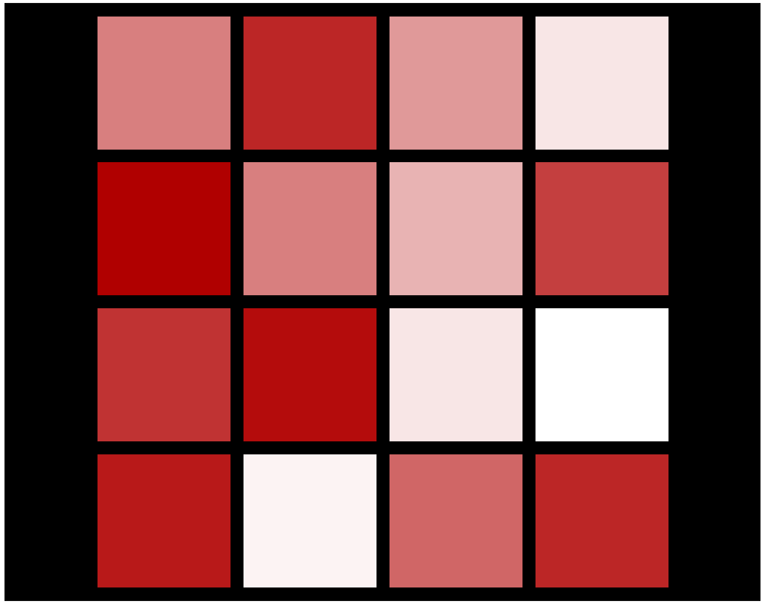
Random Pinning Model

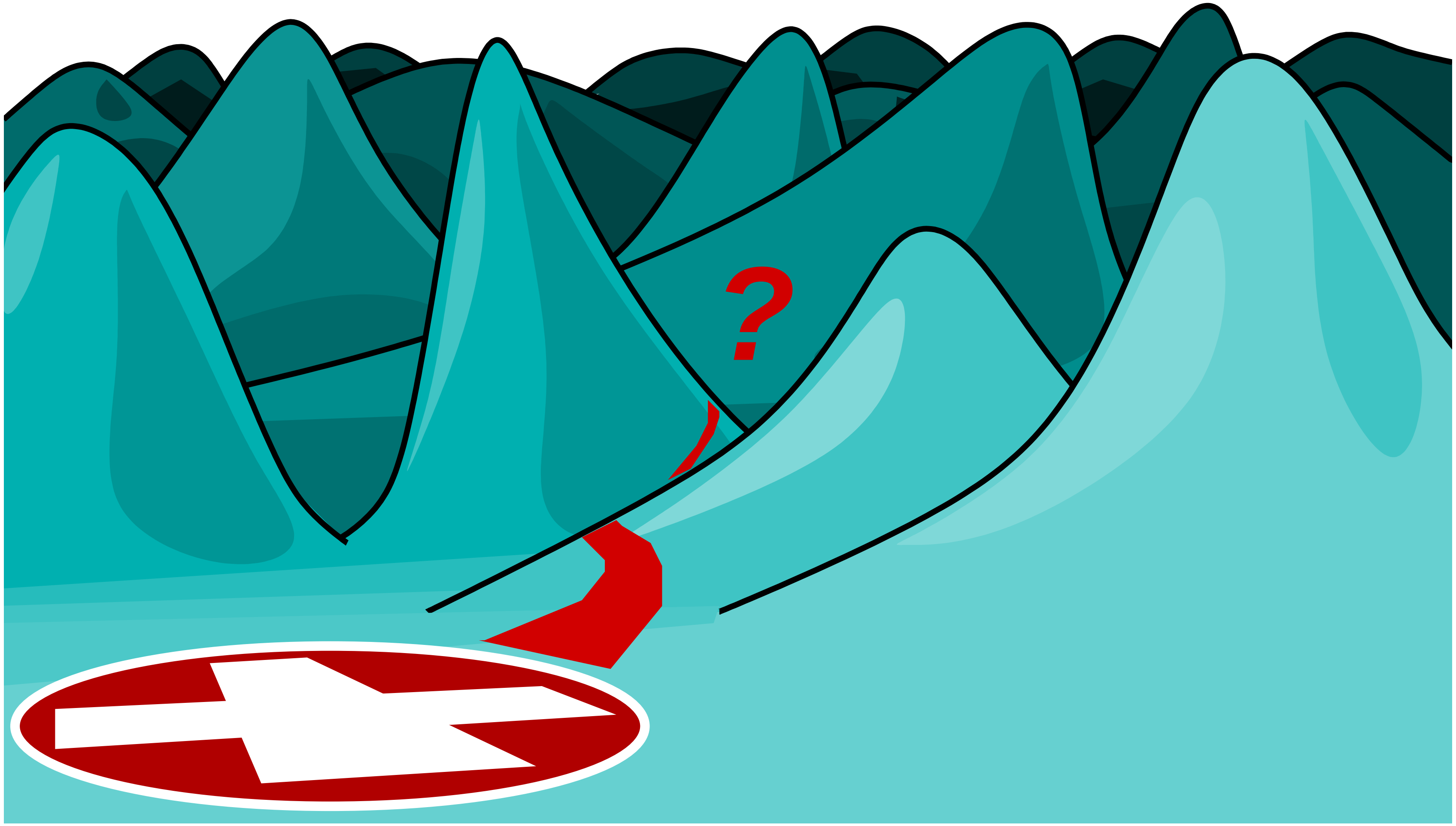
$\mathcal{H} = -\frac{1}{2}\sum_{\mathbf{r}\neq\mathbf{r}'}(q_{\mathbf{r}} - f)\, G(\mathbf{r}-\mathbf{r}')\,(q_{\mathbf{r}'} - f) - \sum_{\mathbf{r}} v_{\mathbf{r}}\, q_{\mathbf{r}}^2.$

$v_{\mathbf{r}} \in [-\pi, \pi)$ is a random variable. The vorticity $q_{\mathbf{r}}$ is restricted to $\{-1, 0, 1\}$.

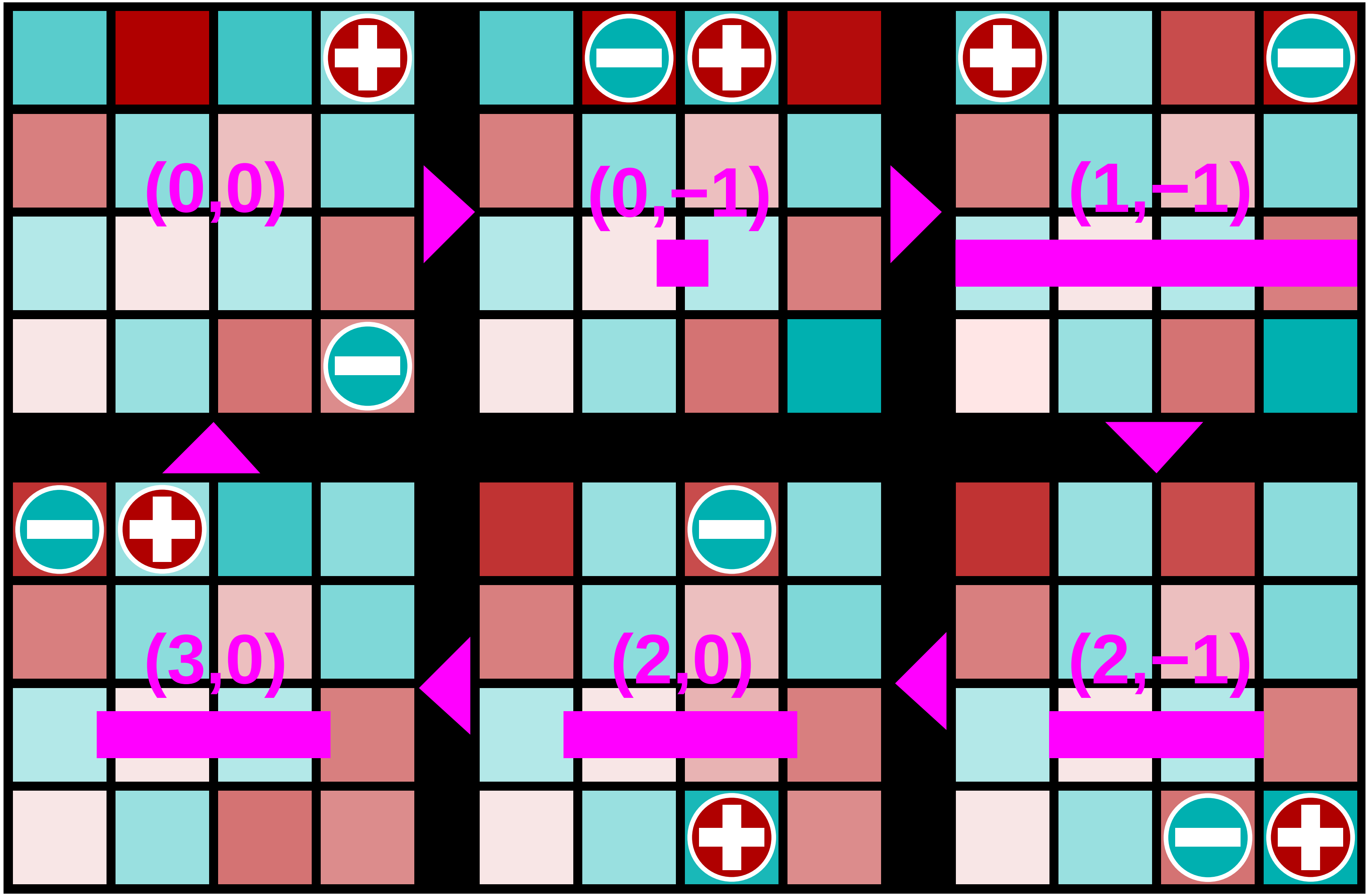
$G(\mathbf{r})$ is the lattice Green's function:

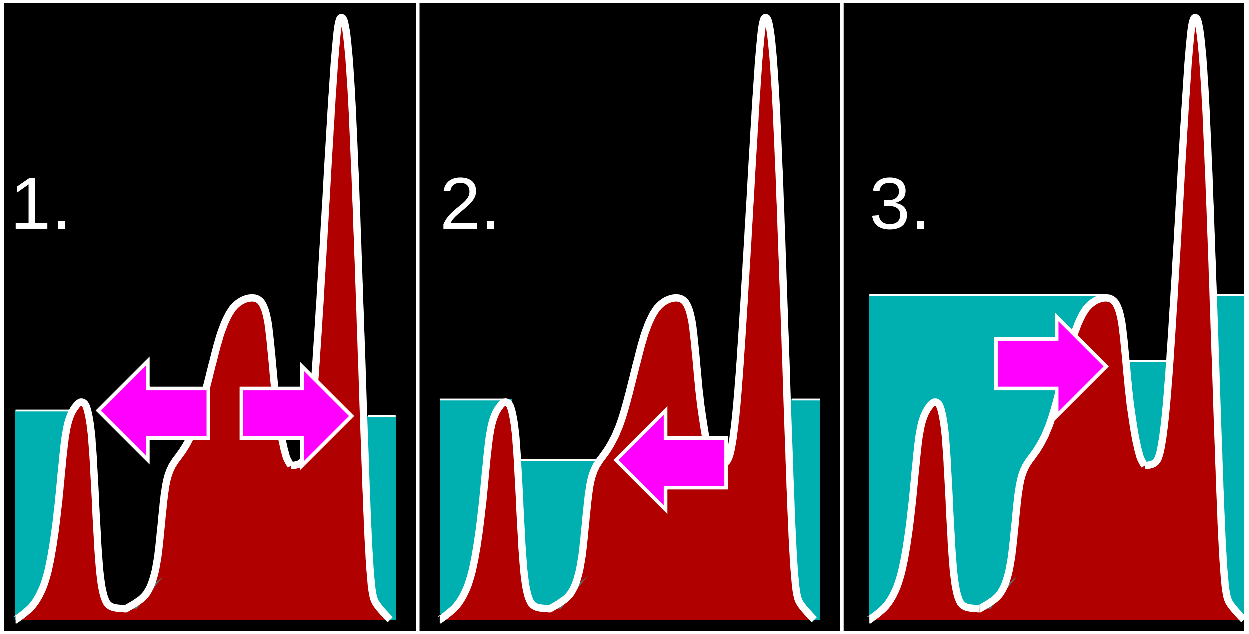
$$G(\mathbf{r}) = \left(\frac{2\pi}{L}\right)^2 \sum_{k\neq 0} \frac{1 - \exp(i\mathbf{k}\cdot\mathbf{r})}{4 - 2\cos k_x - 2\cos k_y}$$



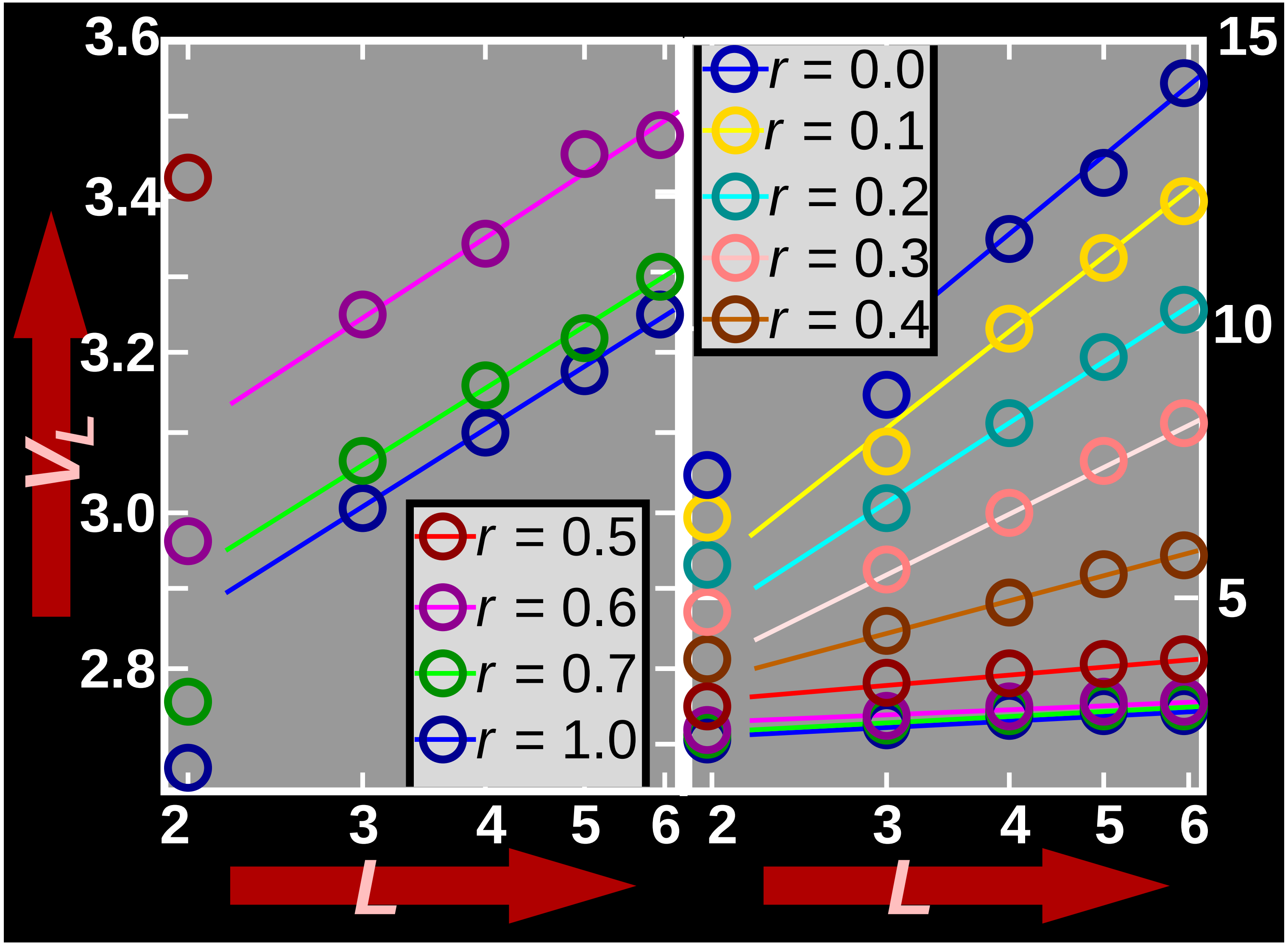


Mobile vortices $\Longrightarrow$ no superconductivity. Are the barriers for vortex motion infinite?

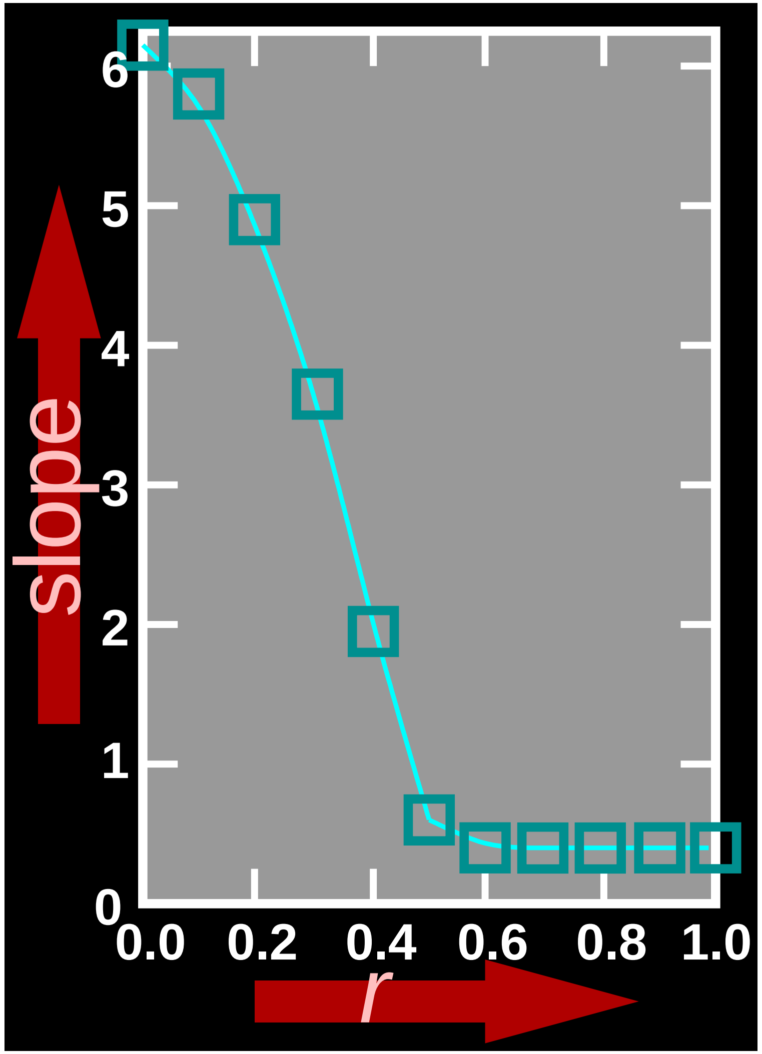

(0,0)
(0,−1)
(1,−1)
(3,0)
(2,0)
(2,−1)

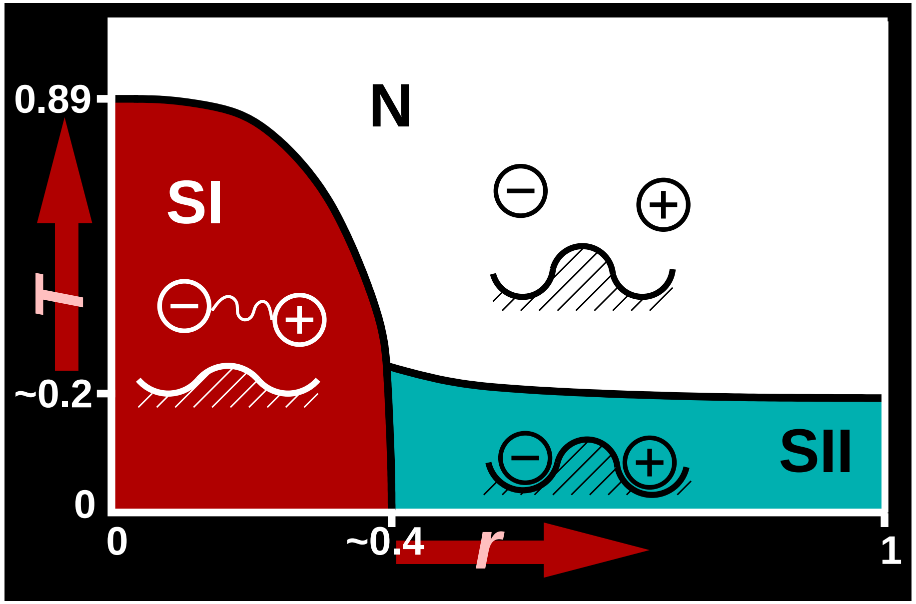


- Generate $4L^2$ configurations by applying the $4L^2$ possible dipole excitations to the current configuration.
- Calculate the energy of each such configuration and put them in list together with their polarization relative to the ground state.
- Take the lowest energy configuration from the list to be the new current configuration.
- If this configuration has already been encountered, but with a different polarization such that $\Delta P = (\pm L, 0)$ we are done. Otherwise, go to the first step.



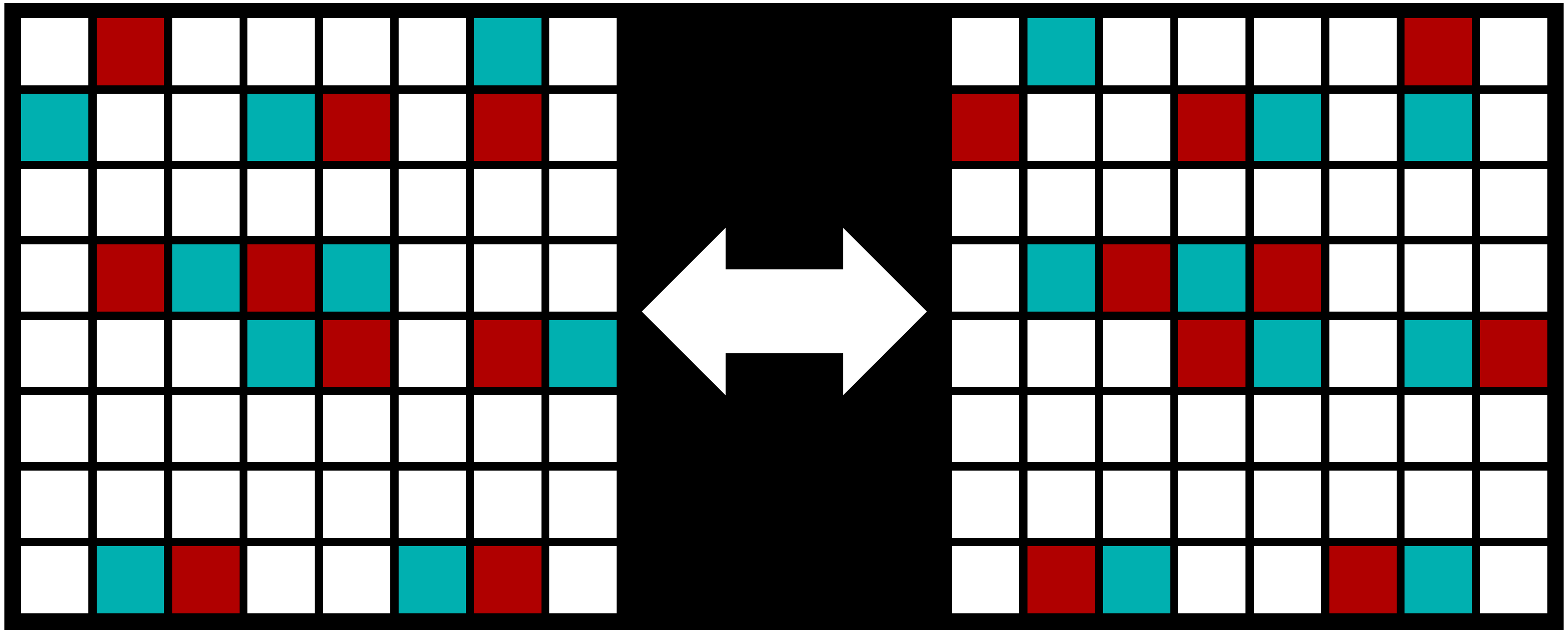
Random Gauge *XY* model



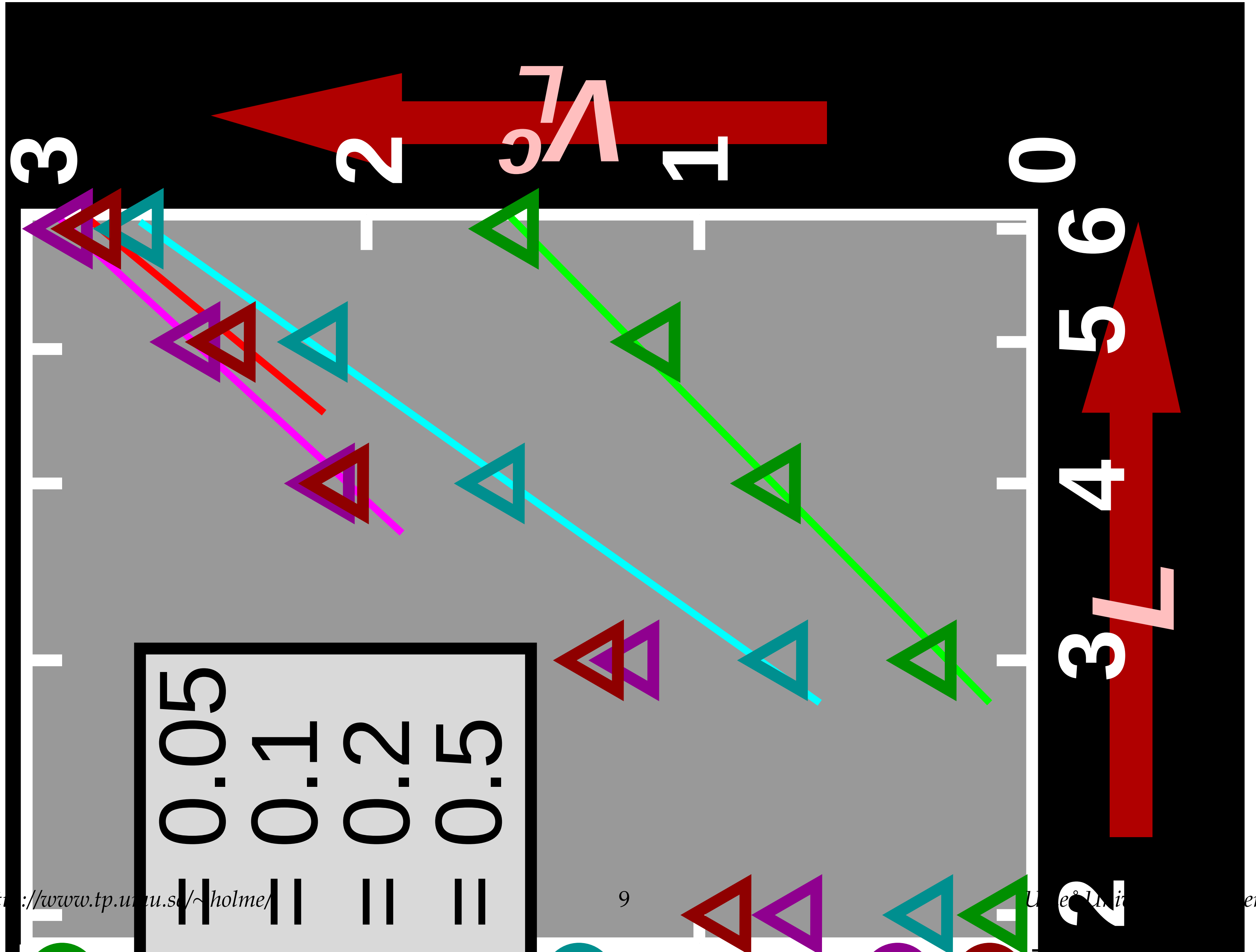


- The slope becomes r-independent for $r > 0.4$.
- A phase boundary has been found at $r \approx 0.4$ earlier.

The Barrier Sustaining Chiral Order

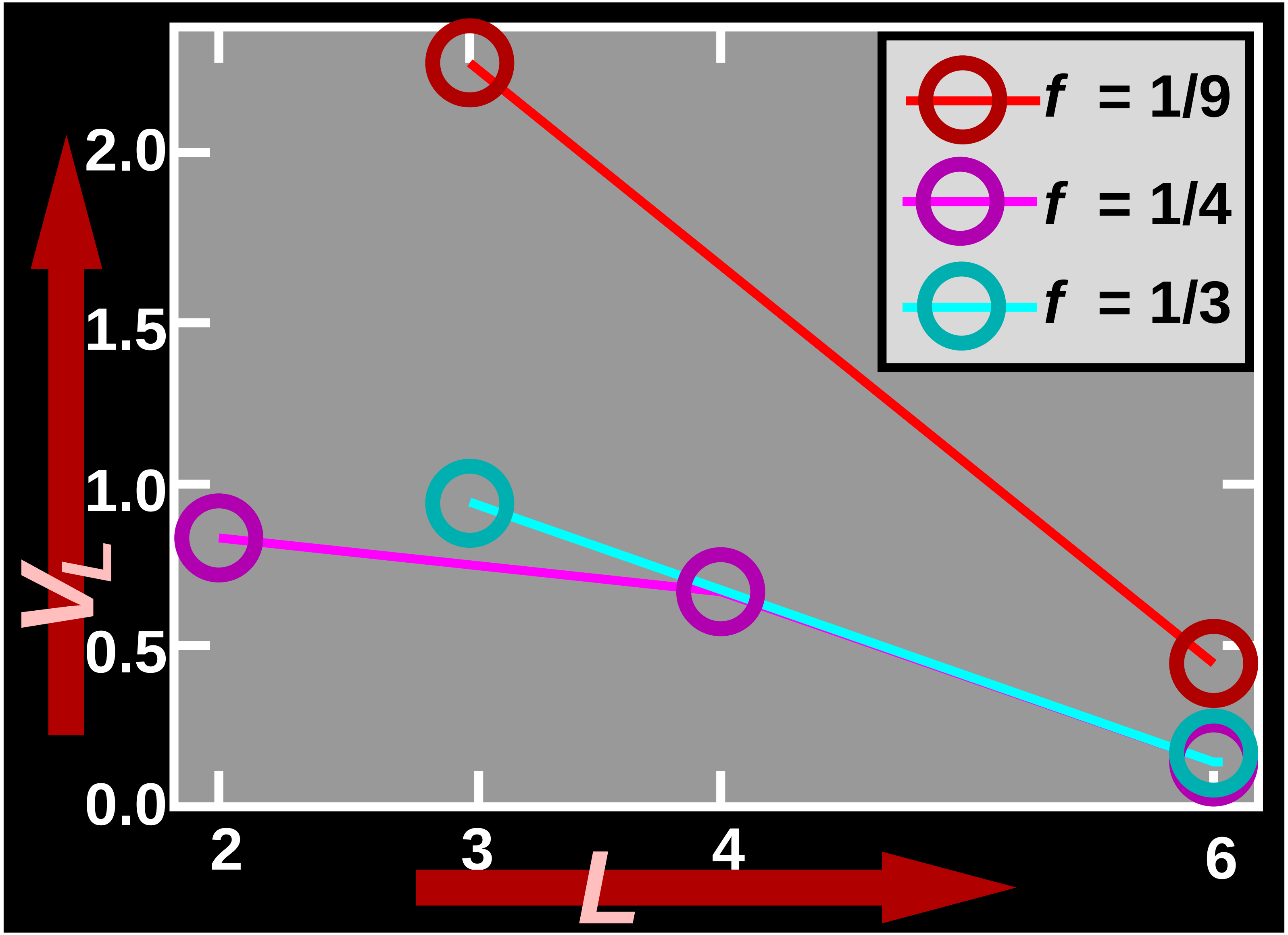


- Generate $4L^2$ configurations by applying the $4L^2$ possible dipole excitations to the current configuration.
- Calculate the energy of each such configuration and put them in list.
- Take the lowest energy configuration from the list to be the new current configuration.
- If this configuration chirally mirrored ground state we are done. Otherwise, go to the first step.



V_c
3
2
1
0
L
6
5
4
3
2
= 0.05
= 0.1
= 0.2
= 0.5

XY Spin Glass model


f = 1/9
f = 1/4
f = 1/3
2.0
1.5
1.0
0.5
0.0
v_L
2
3
4
6
L

Domain Wall Energy

$$\Delta E_{\mathrm{dw}} = \left[\left| \min_{\Delta=0} E - \min_{\Delta=\pi} E \right| \right]$$

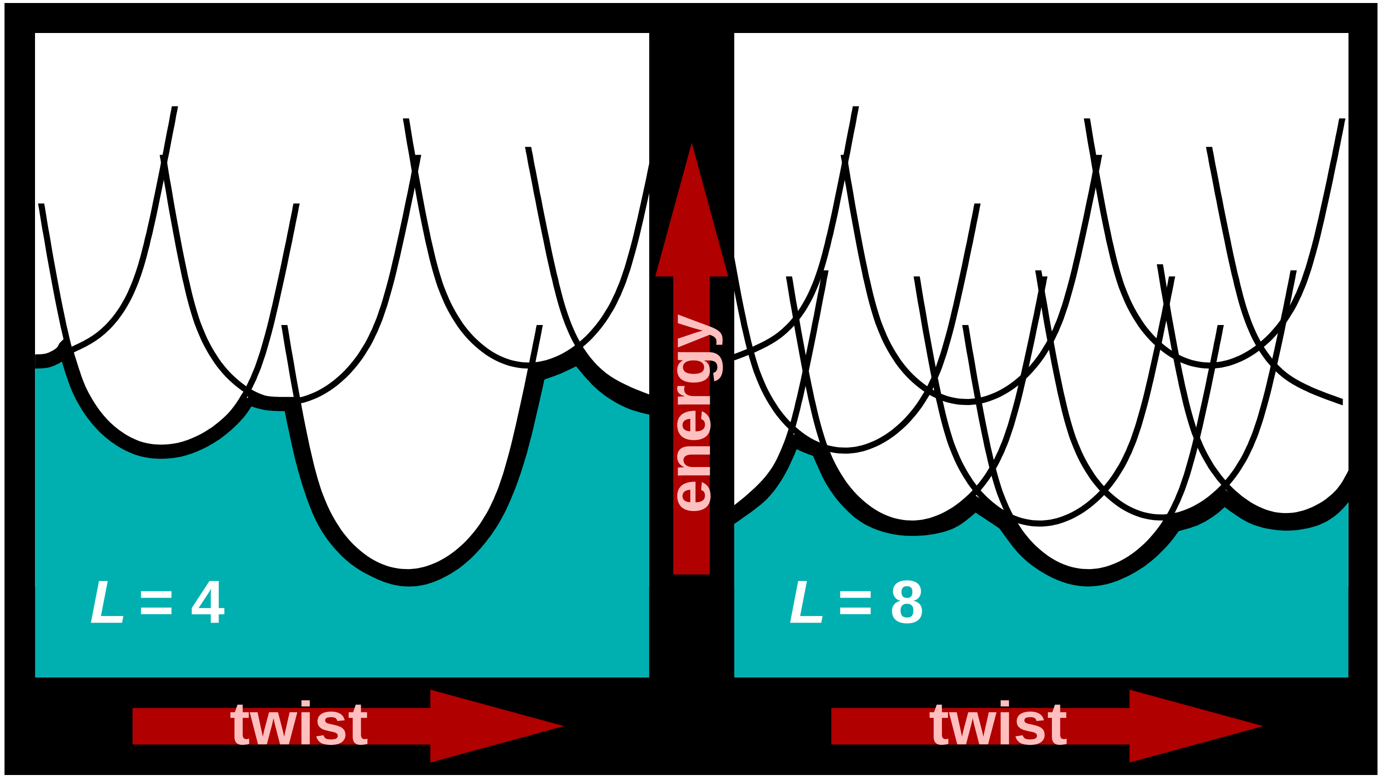
where $[\,\cdot\,]$ marks disorder average.

Best Twist Domain Wall Energy

$$\Delta E_{\mathrm{dw}}^{\mathrm{bt}} = \left[\min_{\Delta=\Delta_0+\pi} E - \min_{\Delta=\Delta_0} E \right]$$

where Δ_0 gives the global twist space ground state.

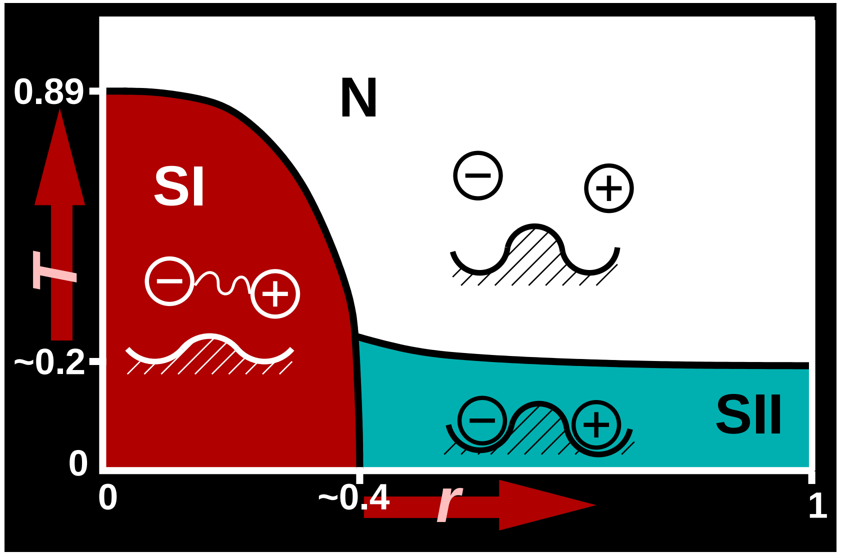
- Both ΔE_{dw} and $\Delta E_{\mathrm{dw}}^{\mathrm{bt}}$ scales like L^{θ}, $\theta < 0$.
- This implies that—provided the system is ergodic—the energy vs twist landscape is flat, and vortices are free to move.
- But . . . $\Delta E_{\mathrm{dw}}^{\mathrm{bt}}$ and ΔE^{bt} measures a barrier against vortex dissipation if and only if the system is ergodic.
- And . . . Ergodicity cannot be verified by the Domain Wall Energy.



- Parables eats up the roughness of the free-energy twist landscape. But . . .
- . . . neighboring points (in the 2D twist space) might be distant in (the L^2-dimensional) phase space.
- If the system is ergodic: $\langle \partial^2 F/\partial\Delta^2|_{\Delta=\Delta_0}\rangle = \Upsilon = 0$
- If ergodicity is broken: $\Upsilon = 1$.

Random Gauge *XY* Model

- There exists a low-T superconducting phase for all values of r.
- In the large-r phase ergodicity is broken.



The *XY* Spin Glass Model

- For almost all s there is no low-T superconducting phase.
- There is a possibility of a chiral phase at low temperatures.

The Random Pinning Model

- There is no low-T superconducting phase.