Licentiate Seminar:

Physics of Two-Dimensional Vortex Glass Models

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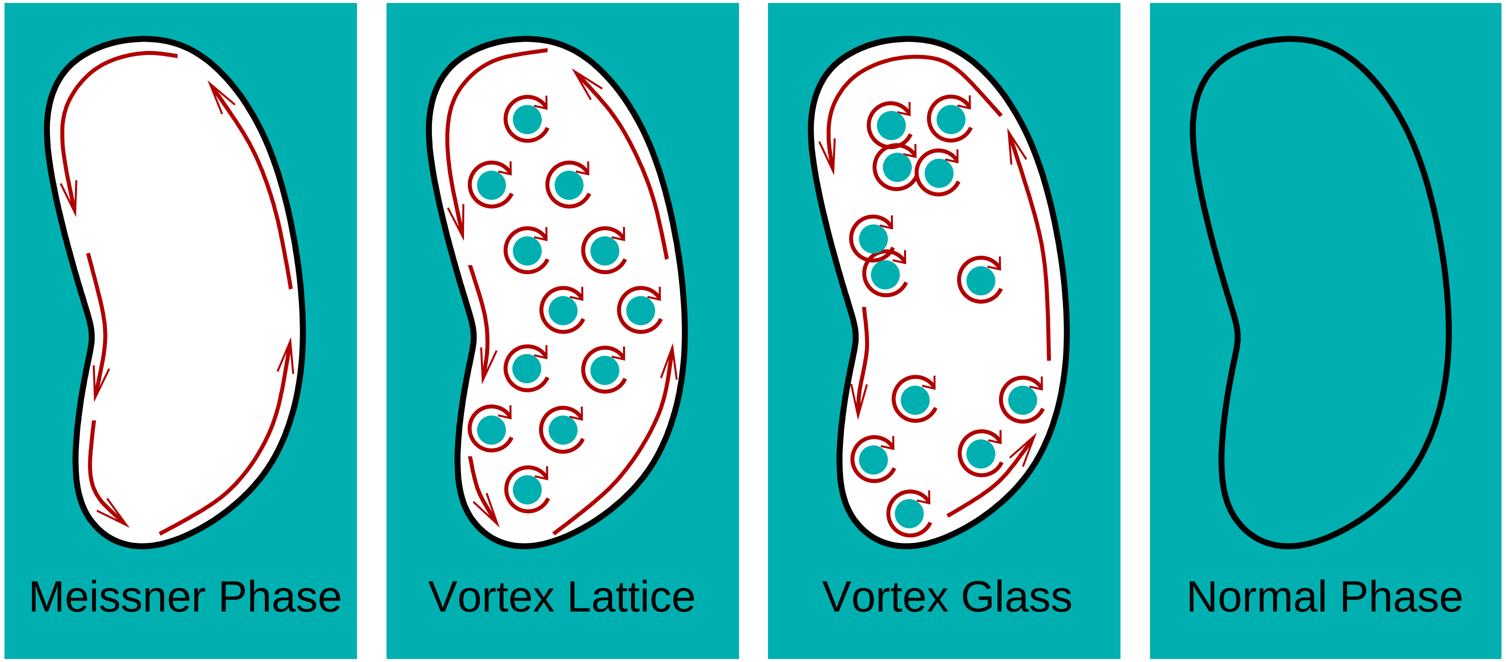
Papers:

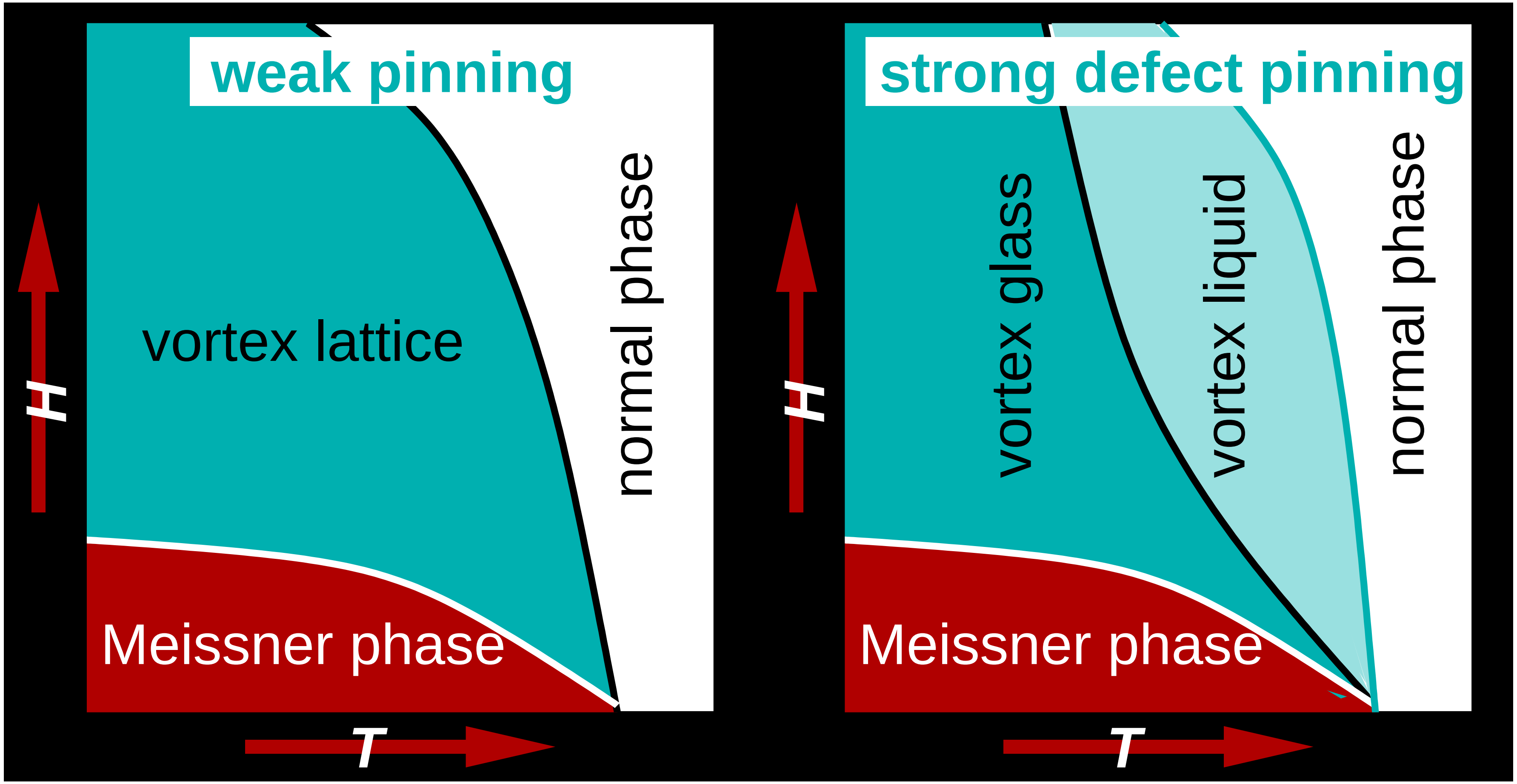
[1.] Petter Holme and Peter Olsson, *A Zero-Temperature Study of Vortex Mobility in Two-Dimensional Vortex Glass Models*.

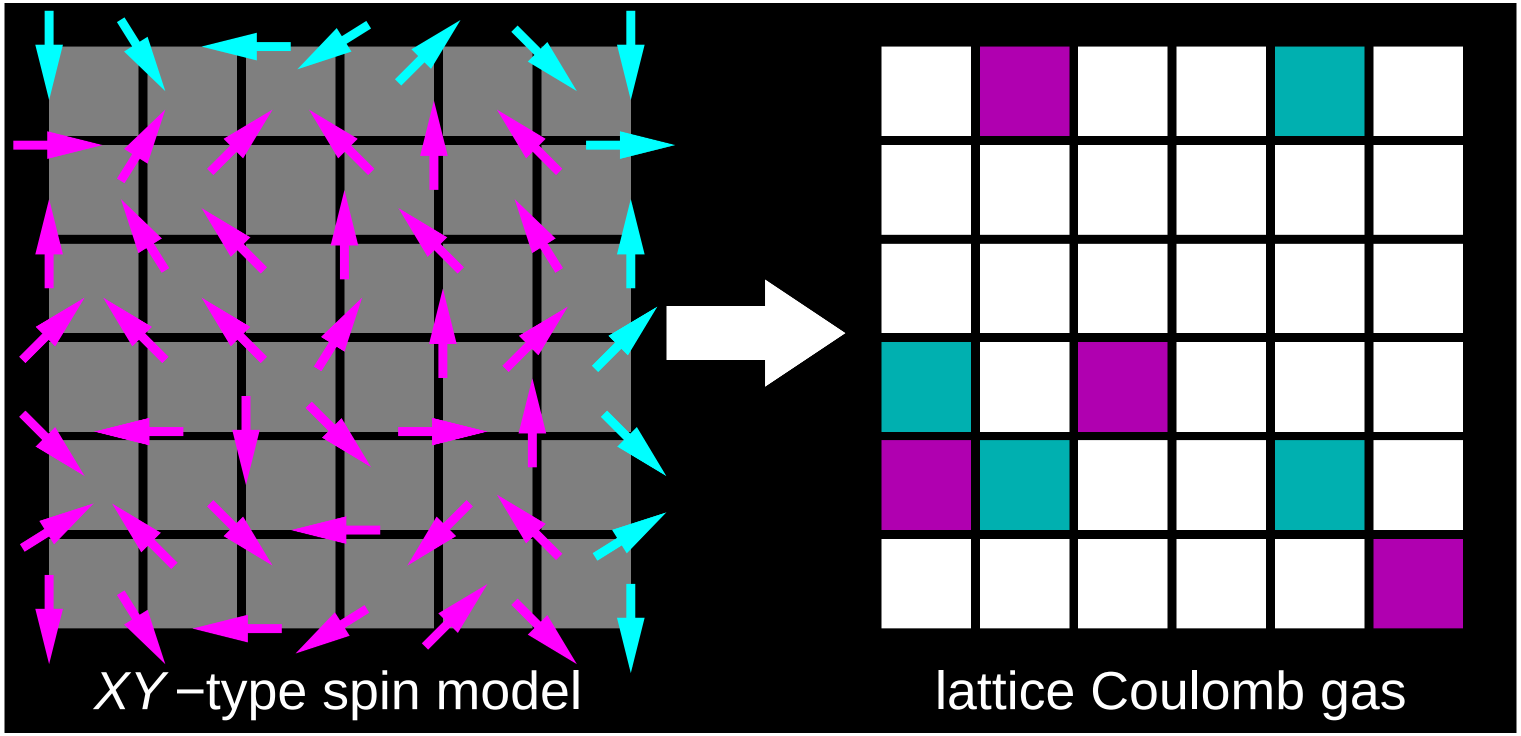
[2.] Petter Holme, Beom Jun Kim, and Petter Minnhagen, *Phase Transitions in the Two-Dimensional Random Gauge XY Model*.

The Vortex Glass Phase

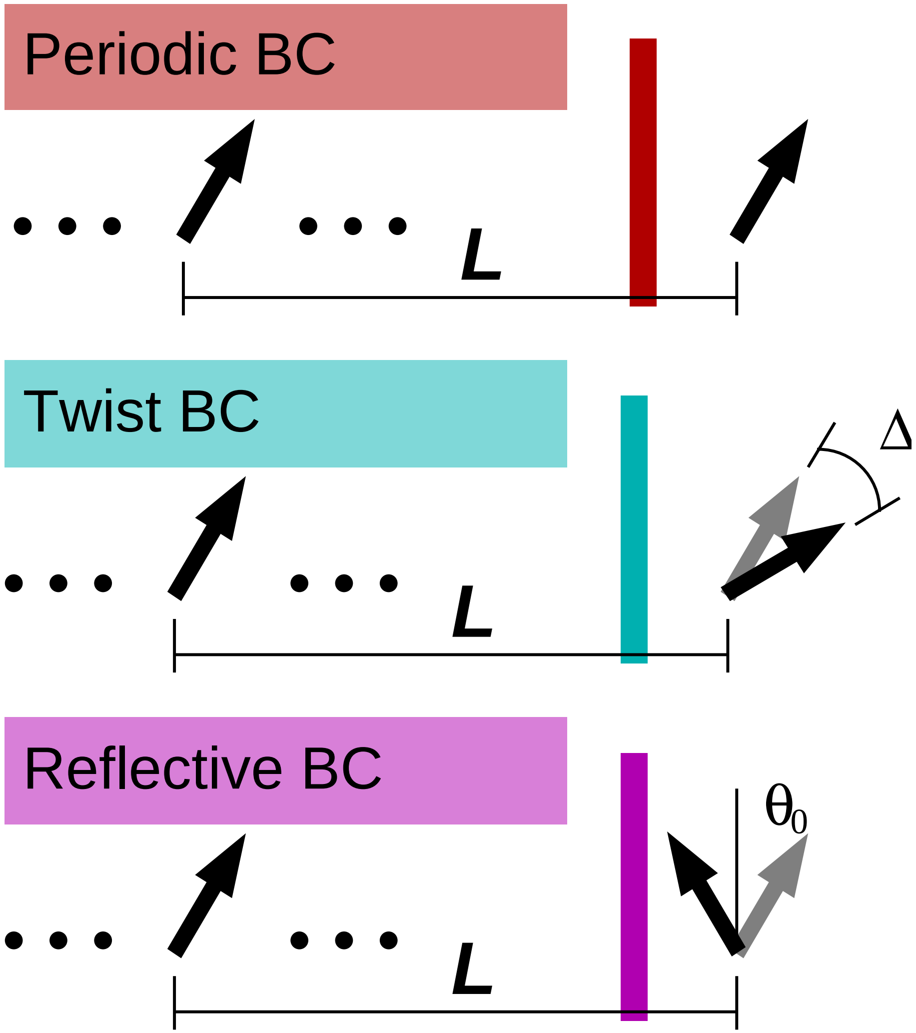


- *Meissner Phase* Magnetic field expelled from the interior by supercurrents close to the surface. Type I and Type II
- *Vortex lattice* Magnetic field penetrates the sample in a regular lattice of vortex-lines. Type II
- *Vortex glass* Magnetic field penetrates the sample in an irregularly distributed vortex-lines. (High-T_c) Type II
- *Normal phase* No supercurrents. Type I and Type II


weak pinning
vortex lattice
normal phase
Meissner phase
H
T
strong defect pinning
vortex glass
vortex liquid
normal phase
Meissner phase
H
T



- In *XY* type spin models there are two types of excitations: *spin waves* and *vortices*.
- Since vortices are considered the more important, spin waves are sometimes removed and the model reduced to a *lattice Coulomb gas*.
- For Zero-*T* methods I will mostly use the CG picture, for Finite-*T* methods the spin picture.



- *Periodic BC:* Every point is interior. Standard for studying bulk properties of a material.
- Provided the BC converge to PBC as $L \to \infty$ it can be modified to study symmetries of the system . . .
- *Twist BC:* For detecting superconductivity (through the helicity modulus Υ).
- *Reflective BC:* For detecting chiral order.

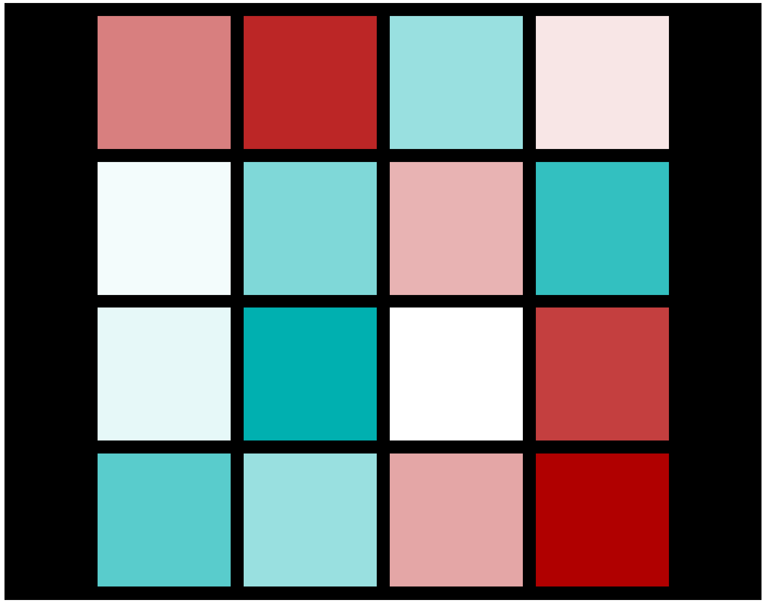
$$\mathcal{H} = -\sum_{(ij)_{\mathrm{nn}}} \cos\left(\theta_i - \theta_j - A_{ij} - \frac{\mathbf{r}_{ij}}{L}\cdot\Delta\right)$$

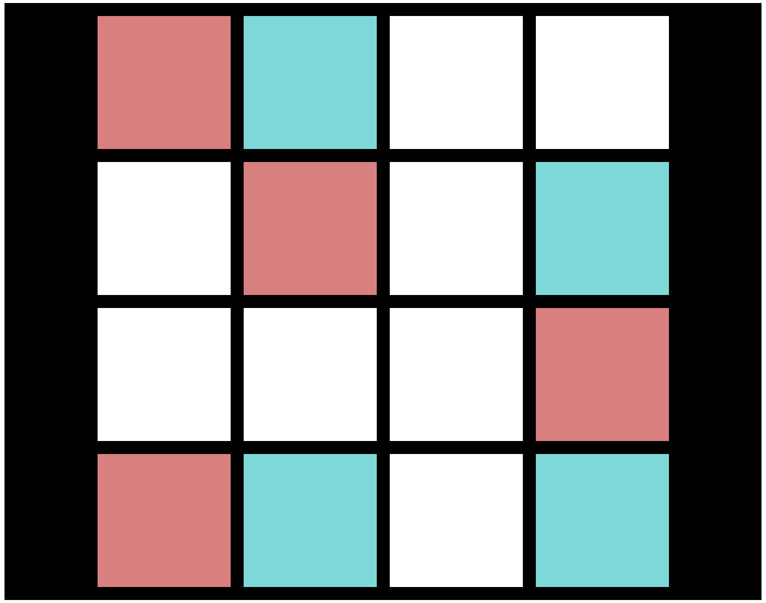
Random Gauge *XY* Model

$A_{ij} \in [-r\pi, r\pi)$, $0 \leqslant r \leqslant 1$. Standard *XY* gauge glass corresponds to $r = 1$.

XY Spin Glass Model

$A_{ij} \in \{0, \pi\}$, $A_{ij} = \pi$ with probability s, $0 \leqslant s \leqslant 1$. Standard *XY* spin glass corresponds to $s = 1/2$.





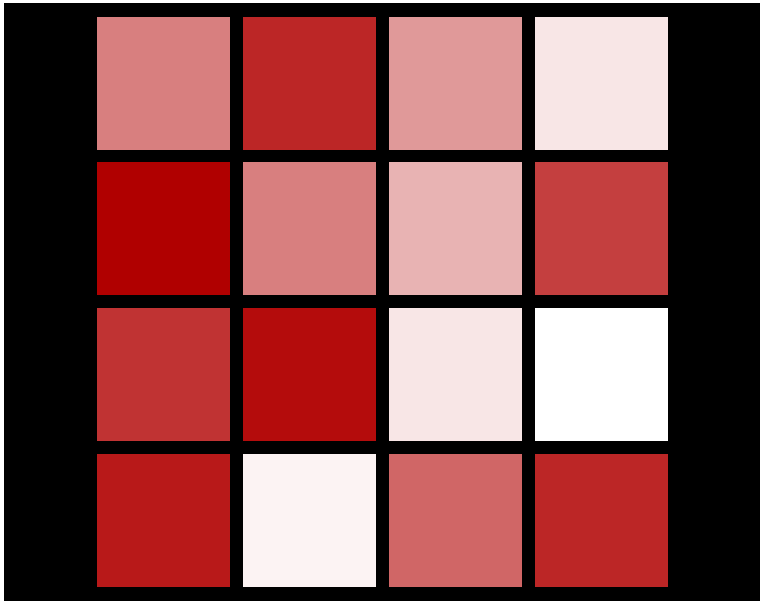
Random Pinning Model

$\mathcal{H} = -\frac{1}{2}\sum_{\mathbf{r}\neq\mathbf{r}'}(q_{\mathbf{r}} - f)\,G(\mathbf{r}-\mathbf{r}')\,(q_{\mathbf{r}'} - f) - \sum_{\mathbf{r}} v_{\mathbf{r}}\, q_{\mathbf{r}}^2.$

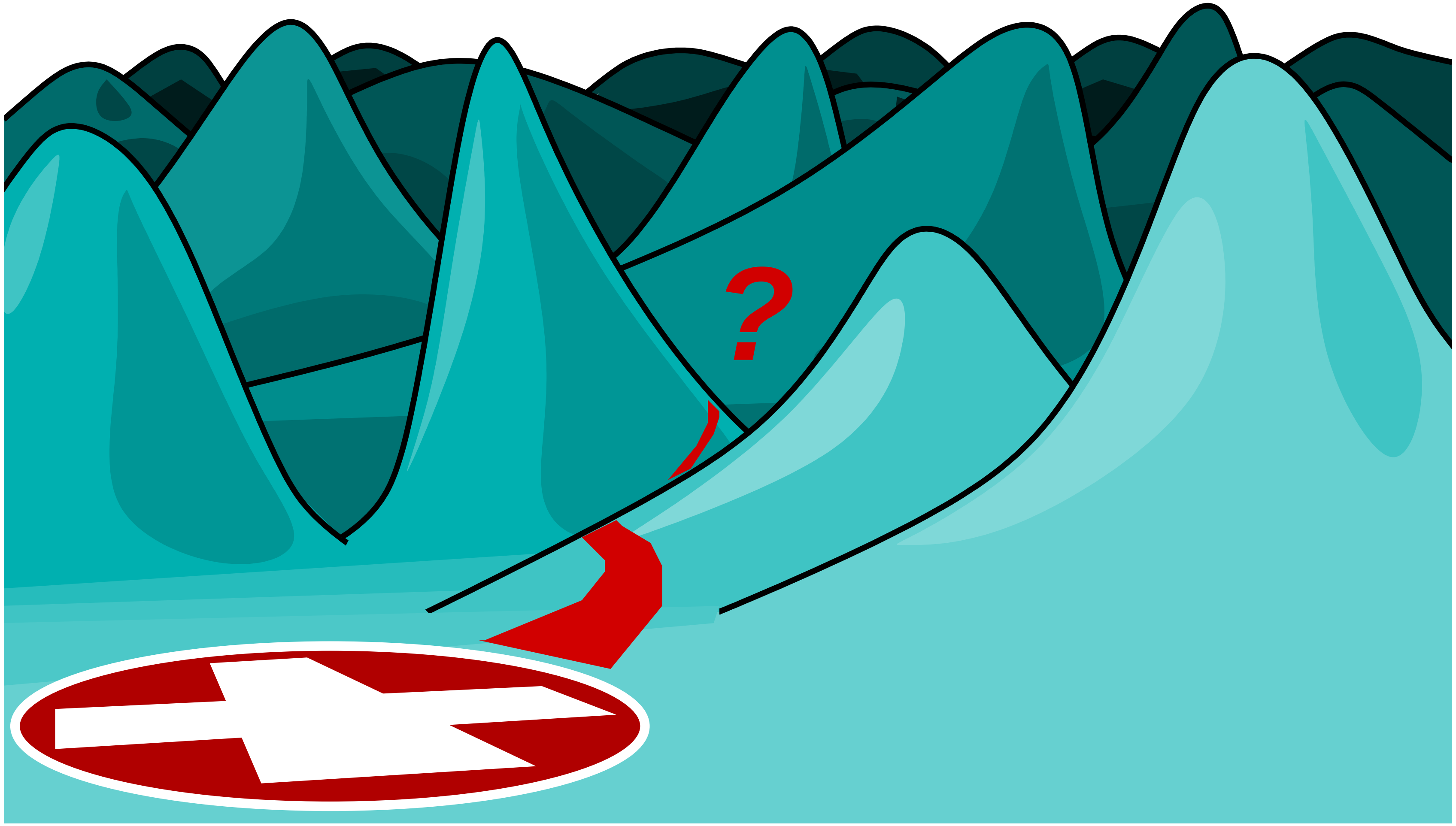
$v_{\mathbf{r}} \in [-\pi, \pi)$ is a random variable. The vorticity $q_{\mathbf{r}}$ is restricted to $\{-1, 0, 1\}$.

$G(\mathbf{r})$ is the lattice Green's function:

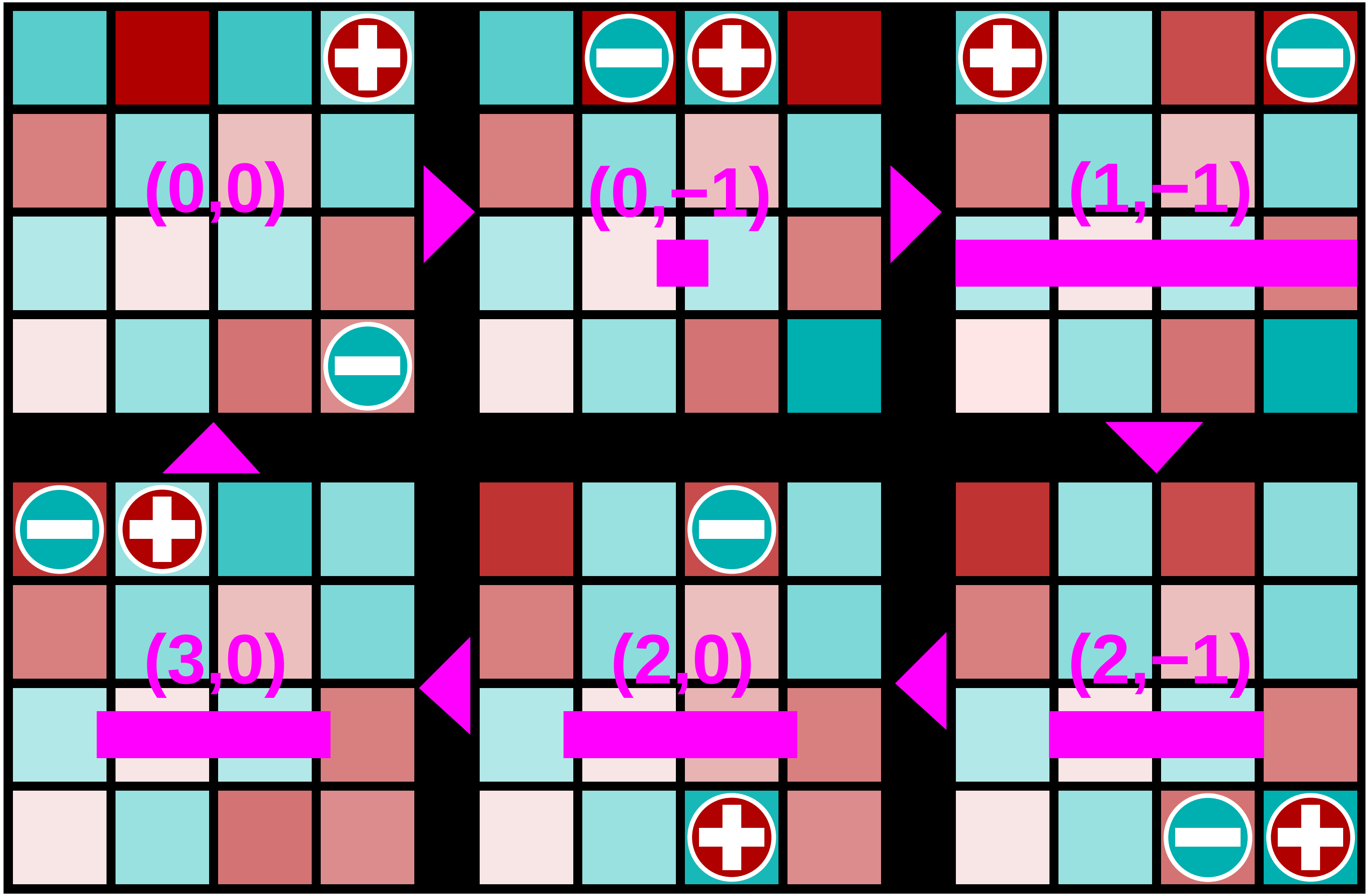
$$G(\mathbf{r}) = \left(\frac{2\pi}{L}\right)^2 \sum_{k\neq 0} \frac{1-\exp(i\mathbf{k}\cdot\mathbf{r})}{4 - 2\cos k_x - 2\cos k_y}$$

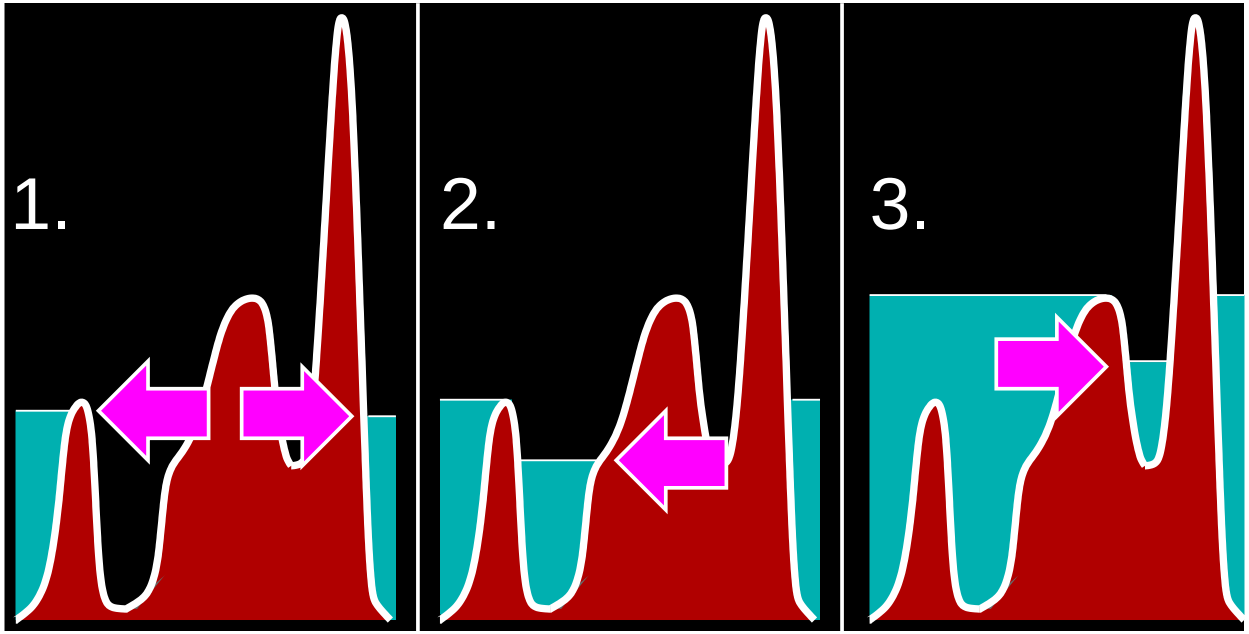


ZERO T: IS VORTICITY TRANSPORT POSSIBLE?

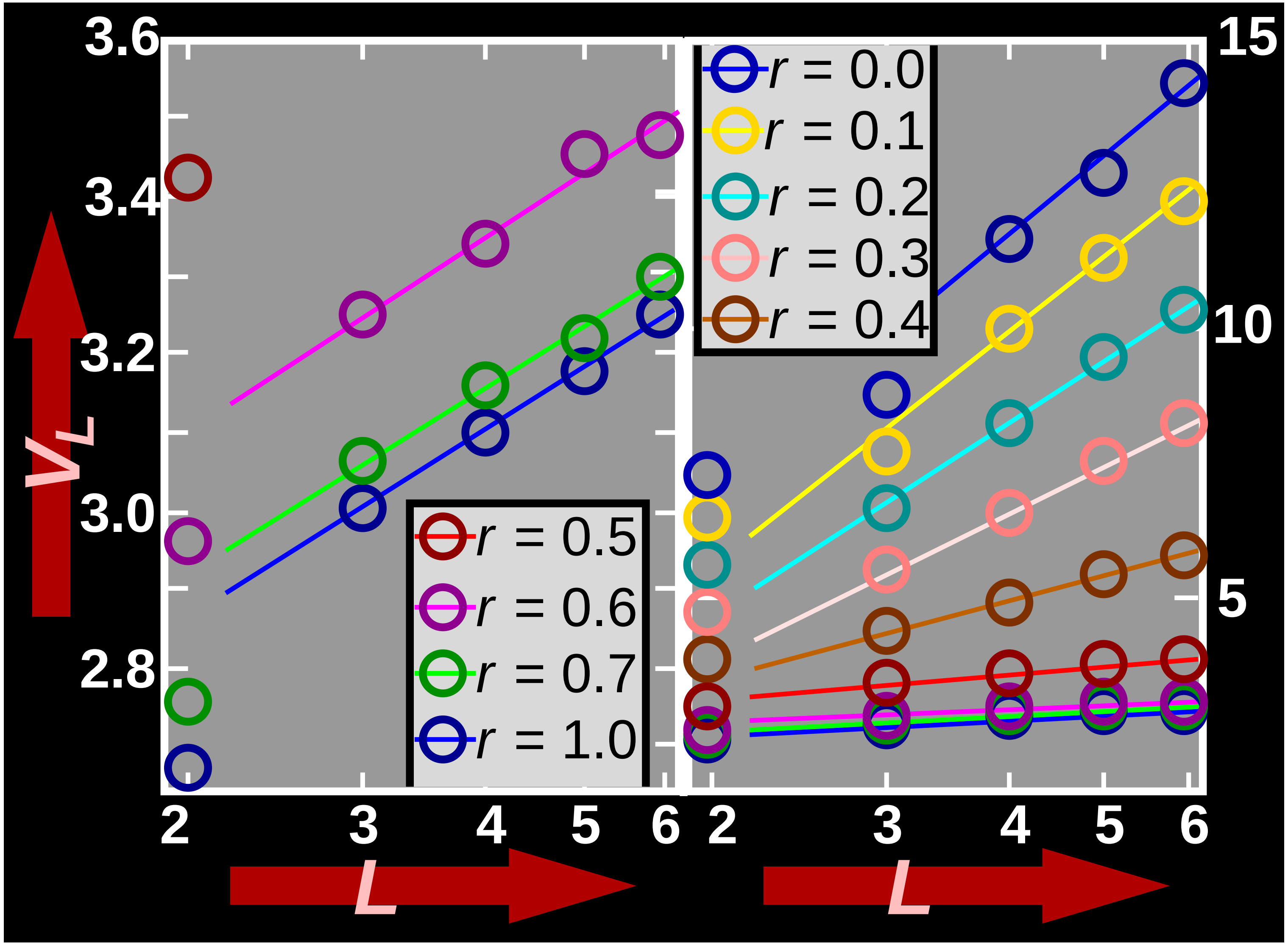


Mobile vortices $\Longrightarrow$ no superconductivity. Are the barriers for vortex motion infinite?

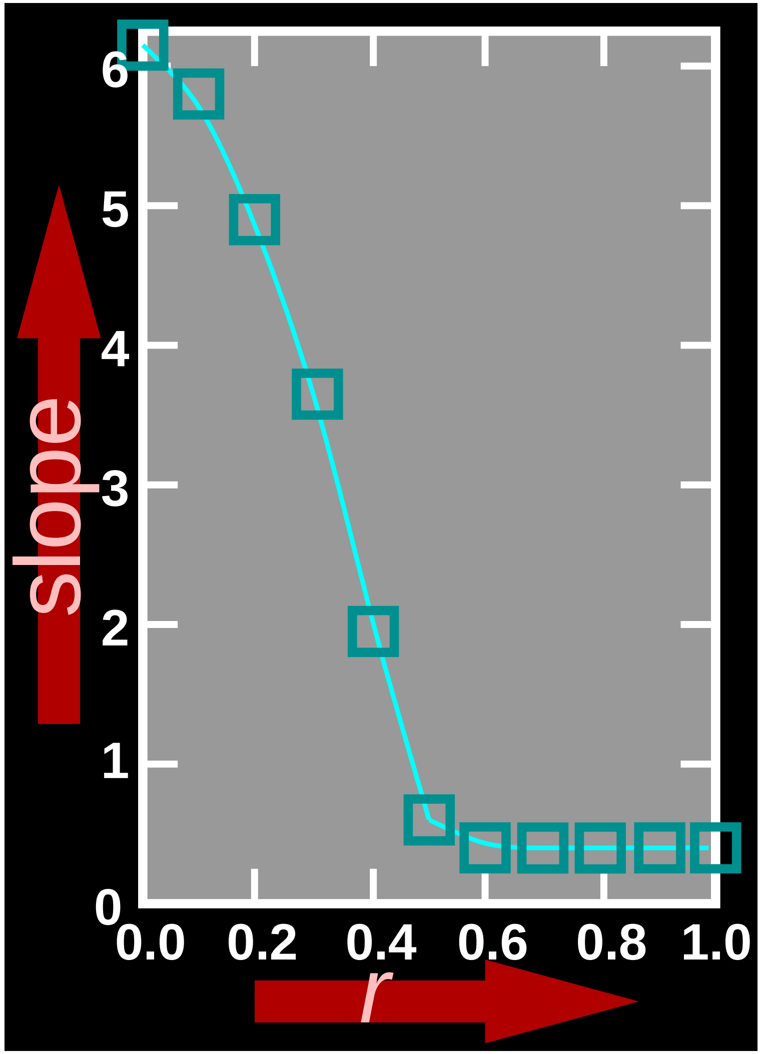

(0,0)
(0,−1)
(1,−1)
(3,0)
(2,0)
(2,−1)

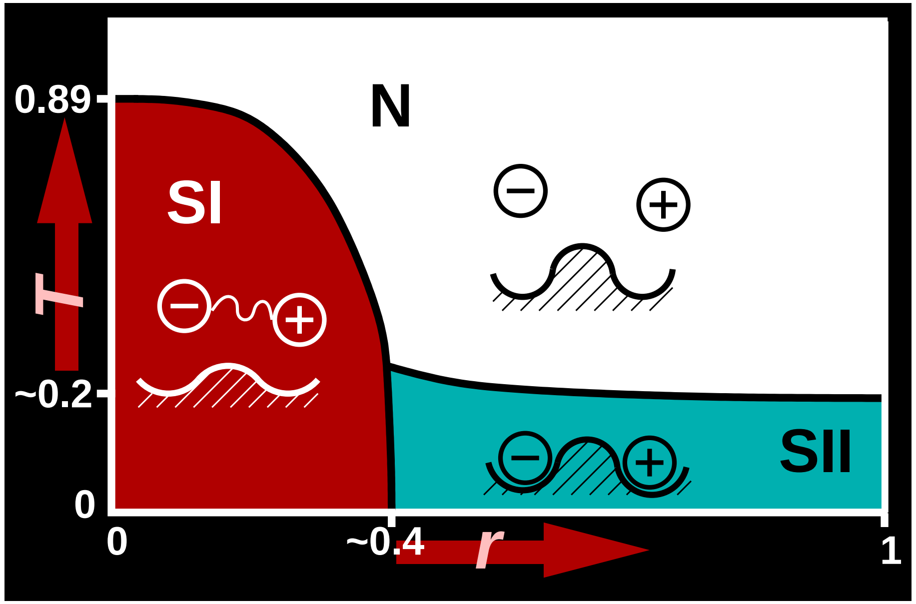


- Generate $4L^2$ configurations by applying the $4L^2$ possible dipole excitations to the current configuration.
- Calculate the energy of each such configuration and put them in list together with their polarization relative to the ground state.
- Take the lowest energy configuration from the list to be the new current configuration.
- If this configuration has already been encountered, but with a different polarization such that $\Delta P = (\pm L, 0)$ we are done. Otherwise, go to the first step.

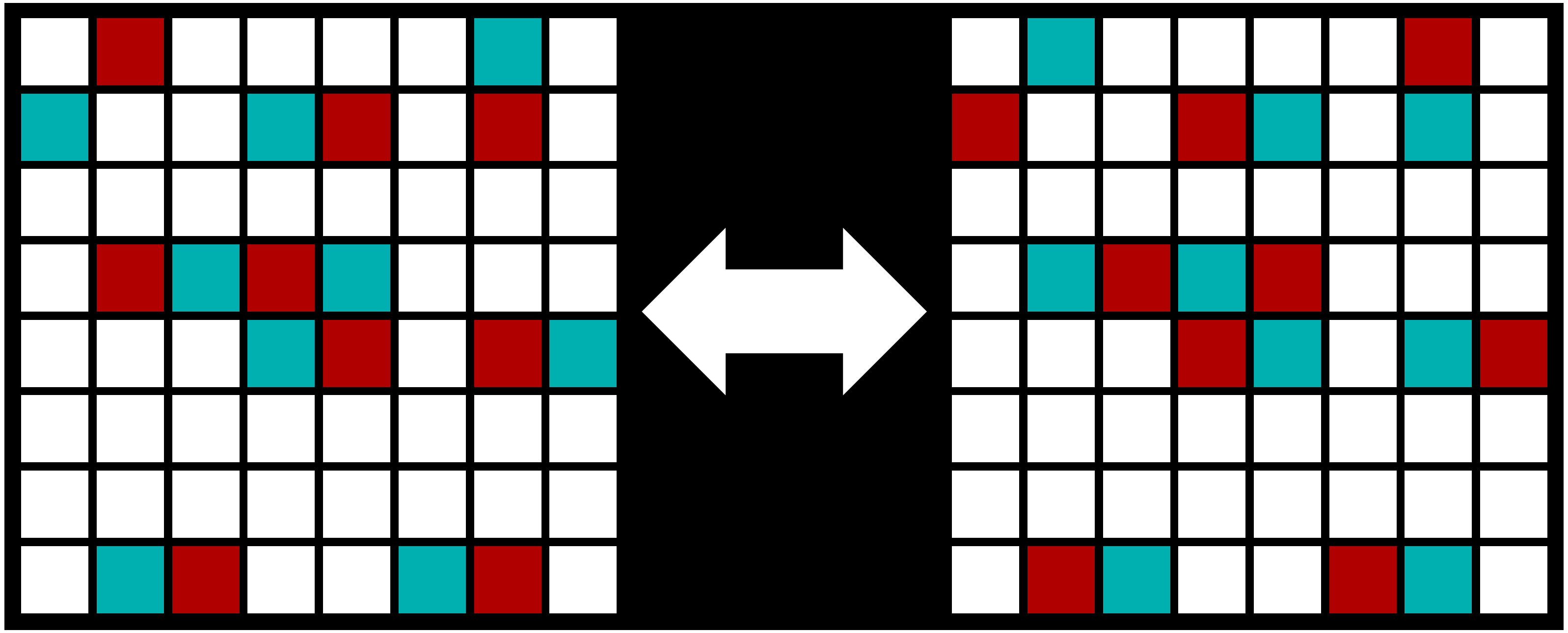


Random Gauge *XY* model

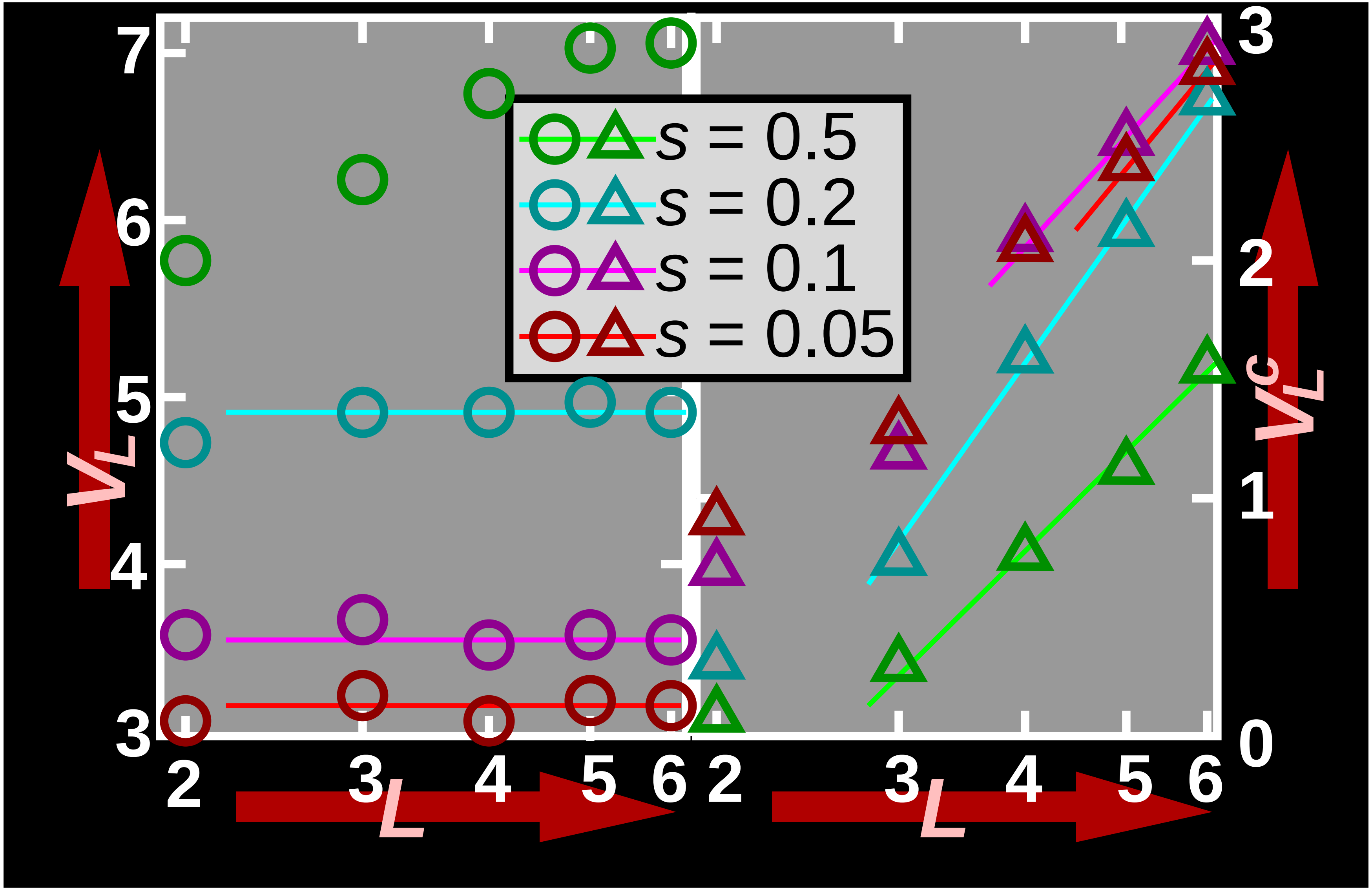




- The slope becomes r-independent for $r > 0.4$.
- A phase boundary has been found at $r \approx 0.4$ earlier.

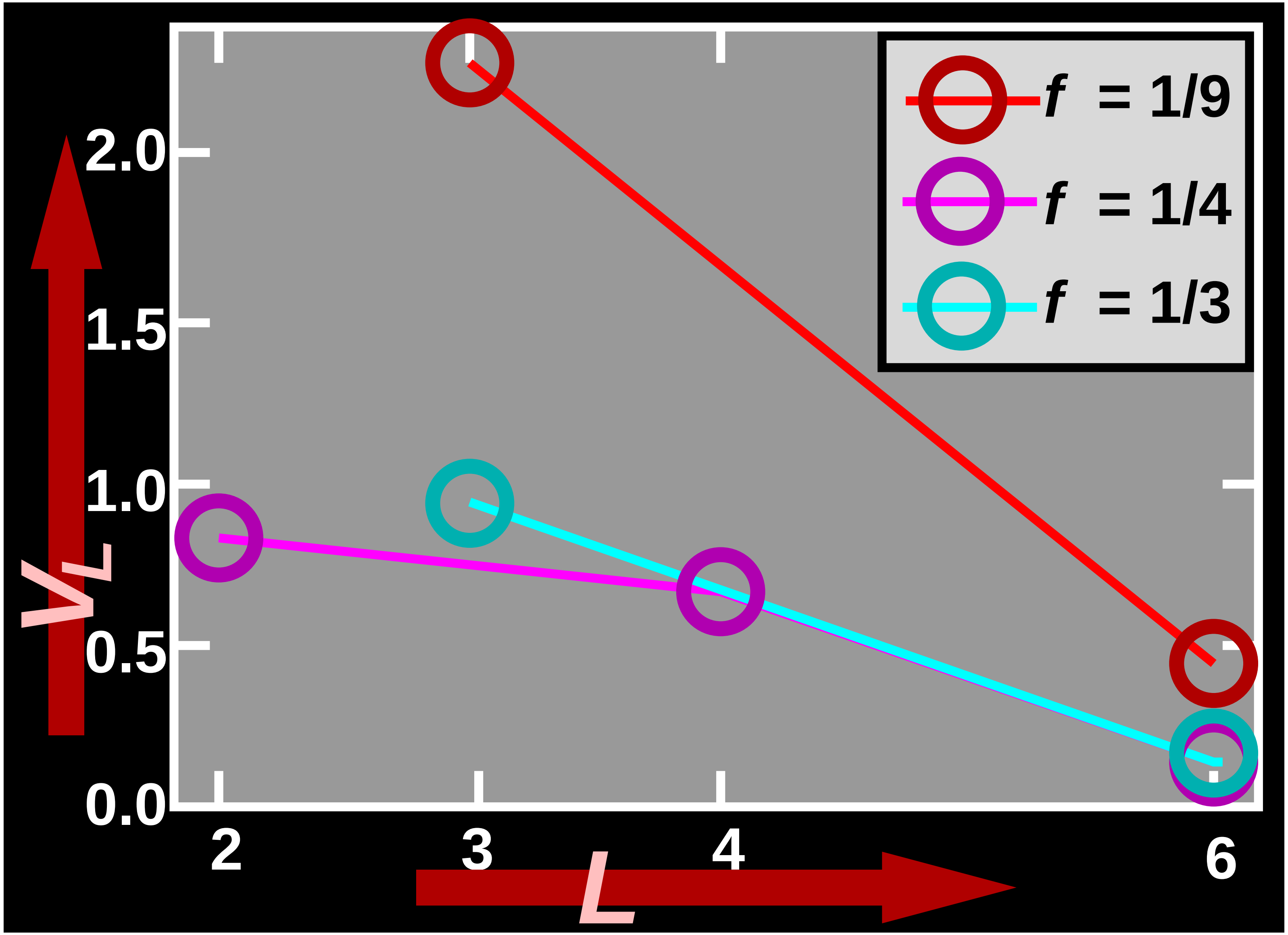


- Generate $4L^2$ configurations by applying the $4L^2$ possible dipole excitations to the current configuration.
- Calculate the energy of each such configuration and put them in list.
- Take the lowest energy configuration from the list to be the new current configuration.
- If this configuration chirally mirrored ground state we are done. Otherwise, go to the first step.



XY Spin Glass model

Zero T: Results for the Random Pinning Model



Domain Wall Energy

$$\Delta E_{\rm dw} = \left[\left| \min_{\Delta=0} E - \min_{\Delta=\pi} E \right| \right]$$

where $[\,\cdot\,]$ marks disorder average.

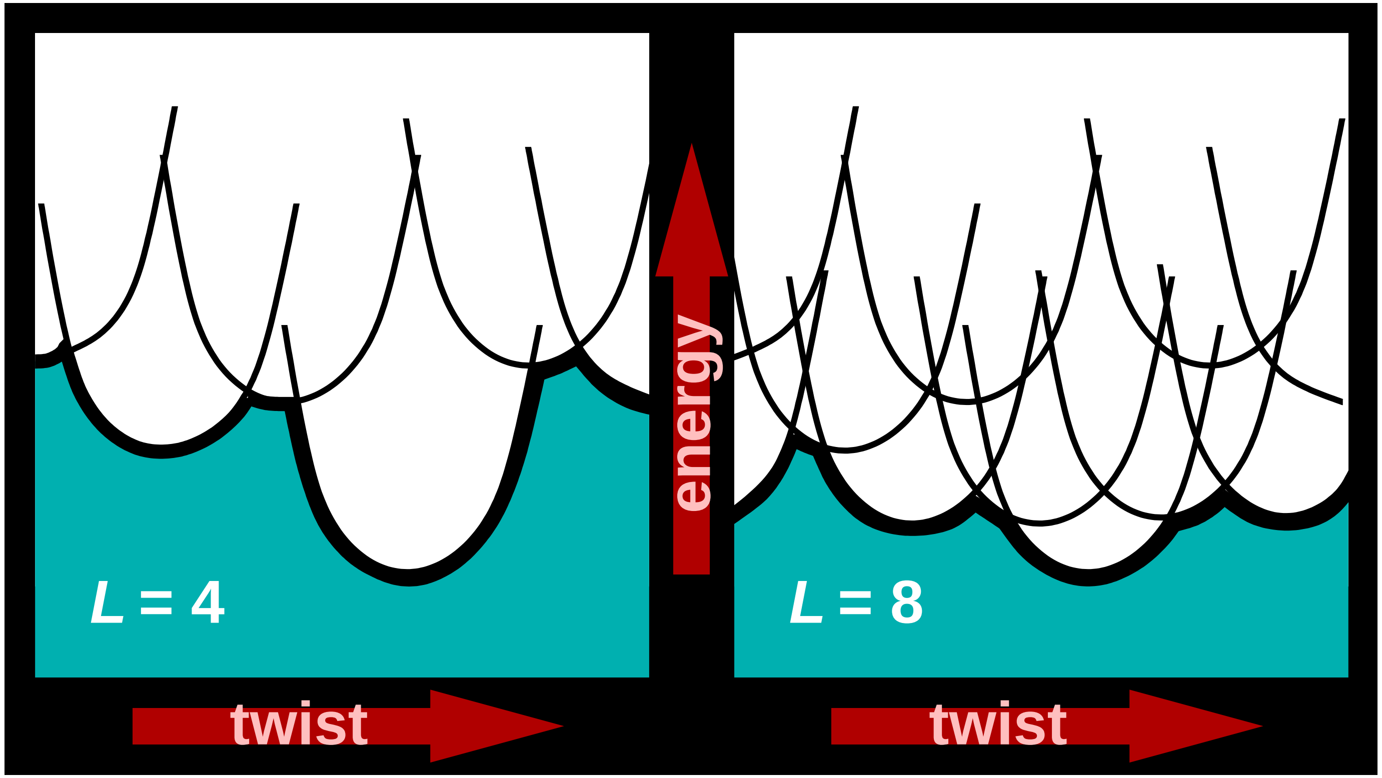
Best Twist Domain Wall Energy

$$\Delta E_{\rm dw}^{\rm bt} = \left[\min_{\Delta=\Delta_0+\pi} E - \min_{\Delta=\Delta_0} E \right]$$

where Δ_0 gives the global twist space ground state.

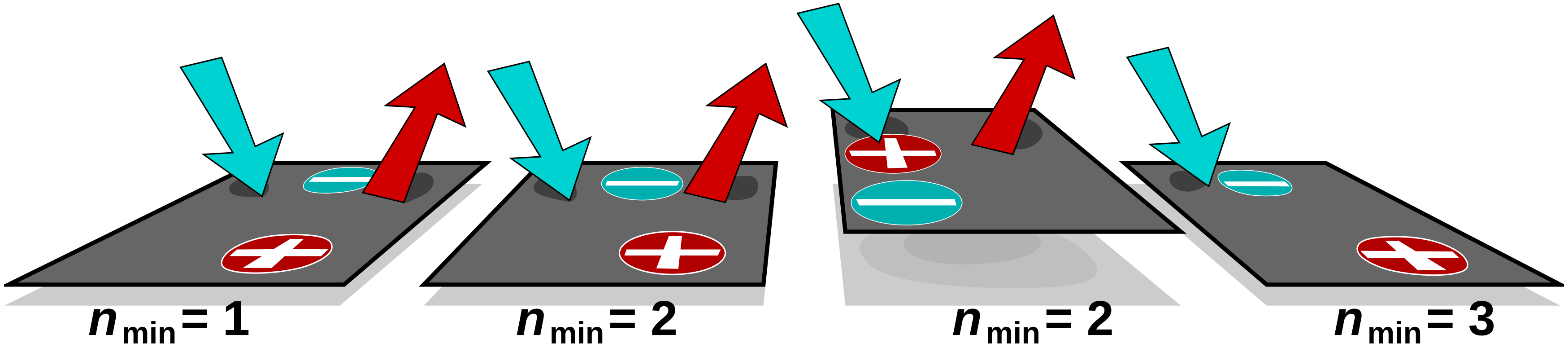
- Both $\Delta E_{\rm dw}$ and $\Delta E_{\rm dw}^{\rm bt}$ scales like L^{θ}, $\theta < 0$.
- This implies that—provided the system is ergodic—the energy vs twist landscape is flat, and vortices are free to move.
- But . . . $\Delta E_{\rm dw}^{\rm bt}$ and $\Delta E^{\rm bt}$ measures a barrier against vortex dissipation if and only if the system is ergodic.
- And . . . Ergodicity cannot be verified by the Domain Wall Energy.

ZERO T: ERGODICITY BREAKING



- Parables eats up the roughness of the free-energy twist landscape. But . . .
- . . . neighboring points (in the 2D twist space) might be distant in (the L^2-dimensional) phase space.
- If the system is ergodic: $\langle \partial^2 F / \partial \Delta^2 |_{\Delta=\Delta_0} \rangle = \Upsilon = 0$
- If ergodicity is broken: $\Upsilon = 1$.

Zero T: Finding the Ground State



The Spin-Quench Algorithm

- 'Heat' the system to $T = \infty$. (= randomize the spin and twist degrees of freedom.)
- 'Cool' fast to $T = 0$. (= decrease the local current until the vortex configuration doesn't change.)
- When the same lowest energy vortex configutation has been re-encountered $n_{\text{min}} = N_{\text{min}}$ times, this is said to be the ground state.

Advantages: Fast for very small systems. Seemingly reliable. Disadvantage: $O(\exp(L))$

For calculating $E_{\text{min}}(\Delta)$ with a fixed Δ (for the Domain Wall Energy), apply the algorithm above to only the spin d.o.f.

- The current:

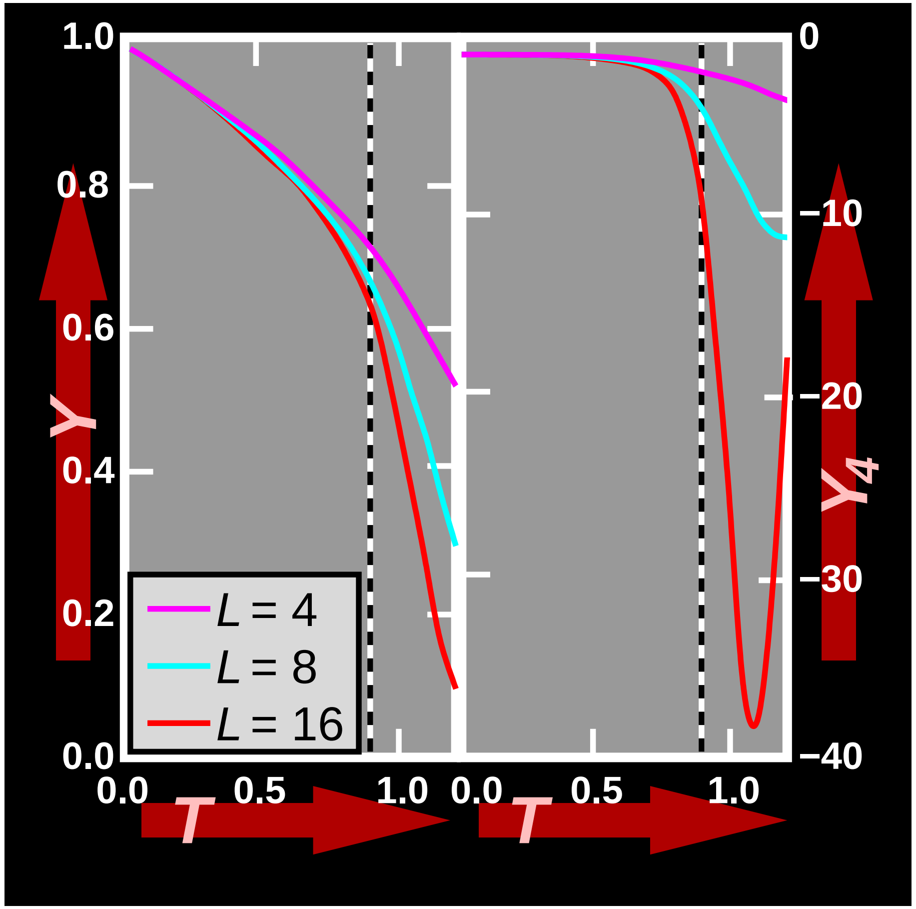
$$\widehat{I} = \left.\frac{\partial F}{\partial \Delta}\right|_{\Delta=\Delta_0}$$

- The helicity modulus

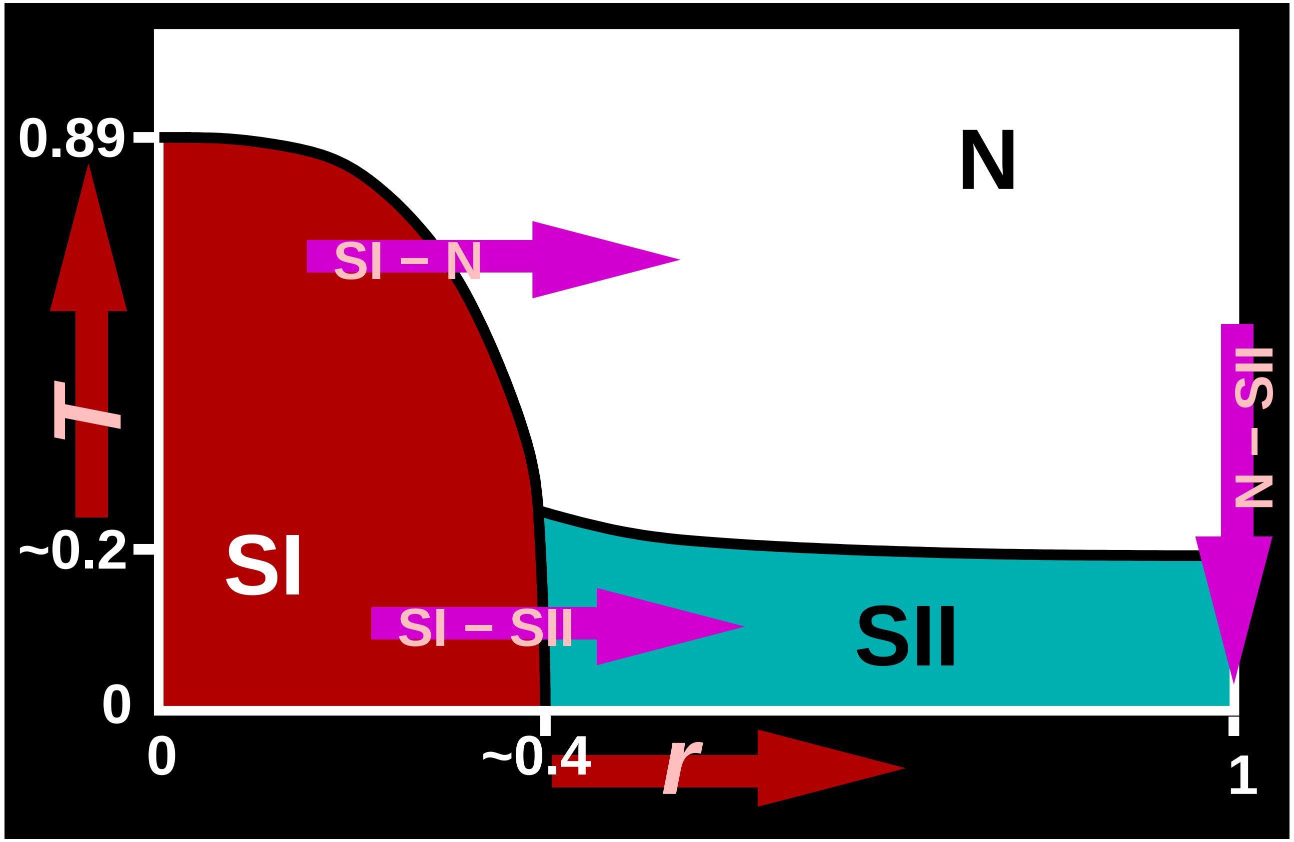
$$\widehat{\Upsilon} = \left.\frac{\partial^2 F}{\partial \Delta^2}\right|_{\Delta=\Delta_0} = -\widehat{E} - \frac{1}{T}(\widehat{I} - I)^2$$

- The fourth order modulus:

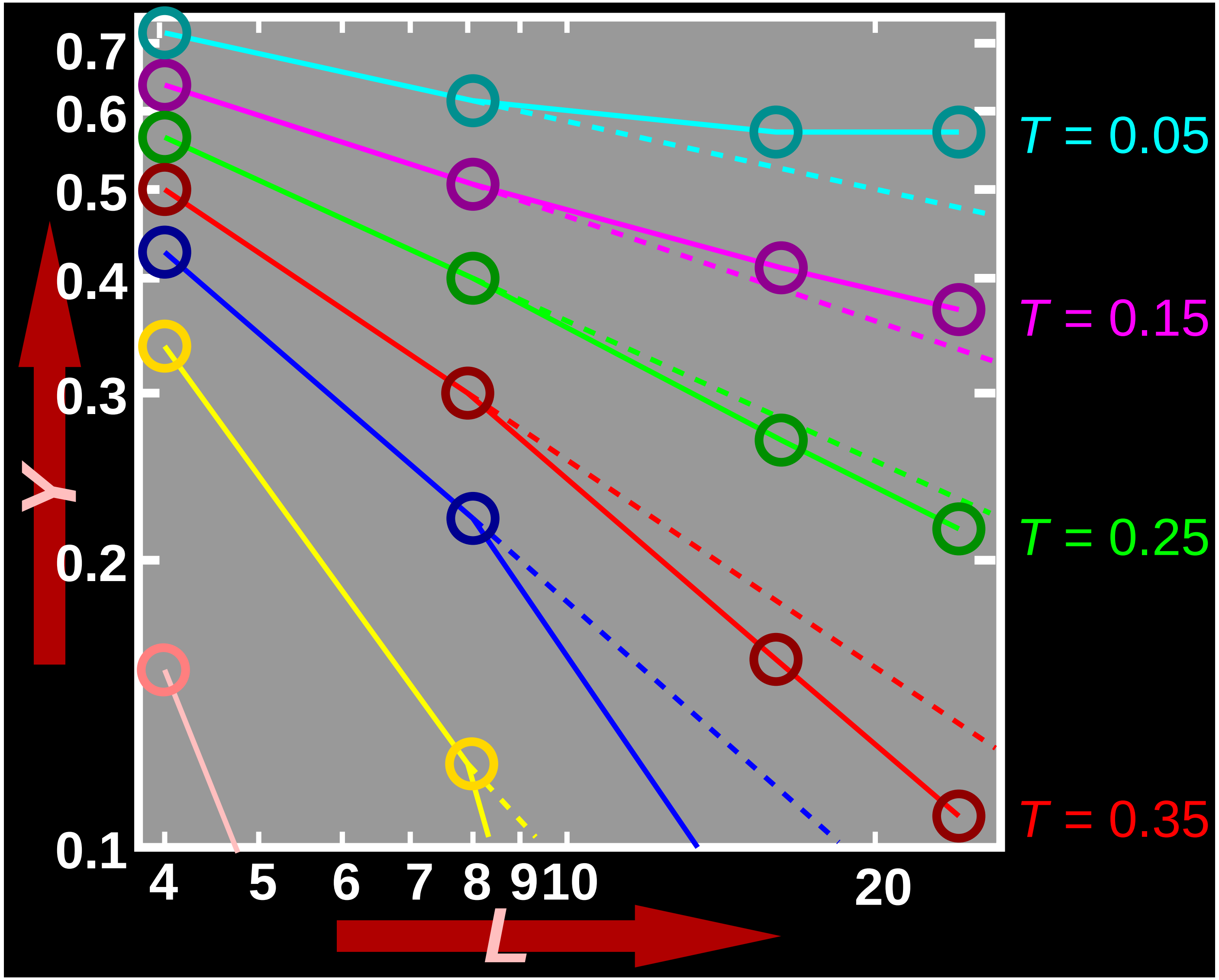
$$\widehat{\Upsilon}_4 = \left.\frac{\partial^4 F}{\partial \Delta^4}\right|_{\Delta=\Delta_0} = -4\Upsilon - 3\widehat{E} - \frac{3L^2}{T}(\widehat{\Upsilon} - \Upsilon)^2 + \frac{2}{L^2T^3}(\widehat{I} - I)^4$$



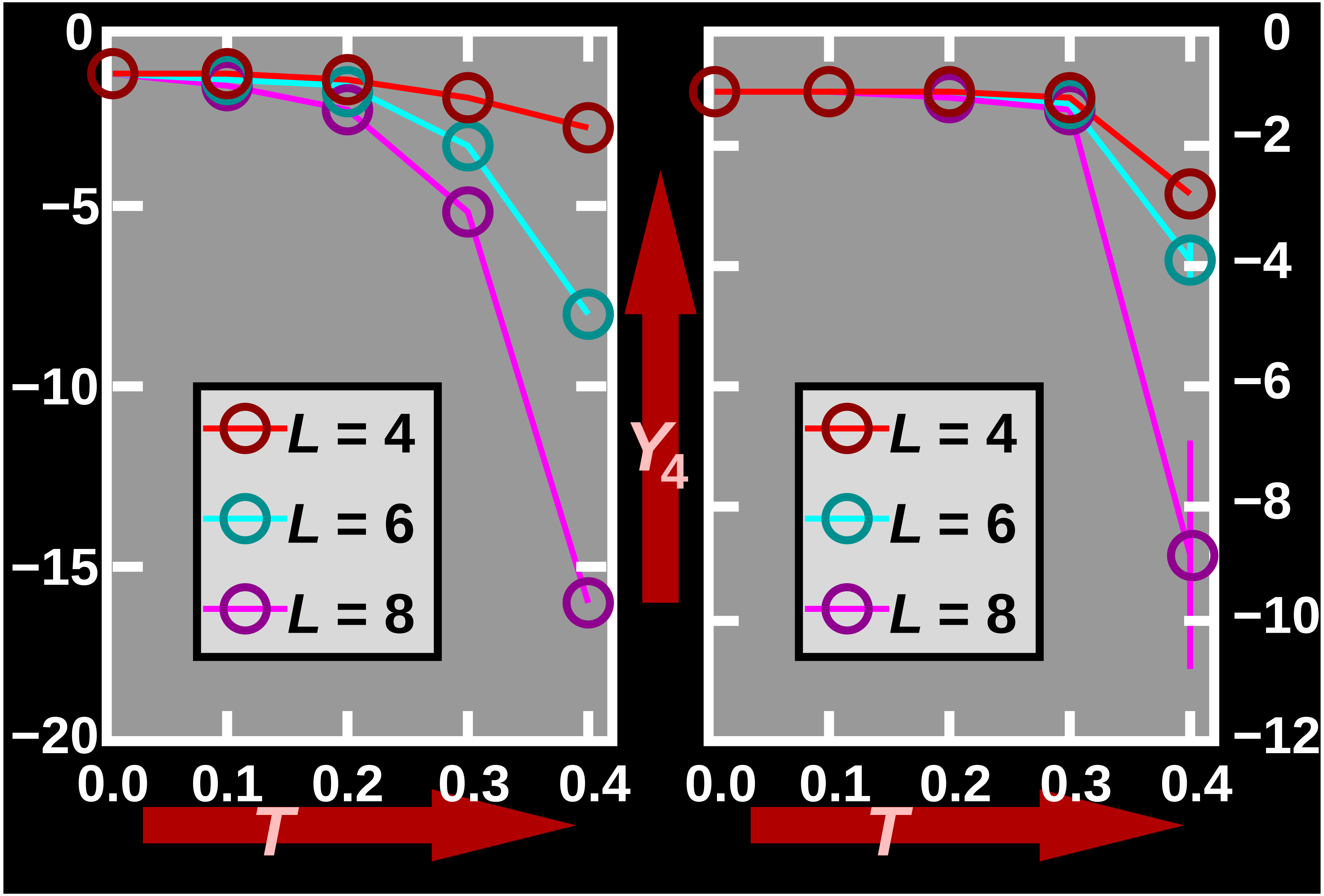
Finite T: How to See the Three Transitions



- The N – SII transition is located to the temperature where $\Upsilon(T, L) > 0$ as $L \to \infty$.
- For the SI – SII transition, the Υ signal is too weak so we use Υ_4.
- The N – SI transition has earlier been located by finite T Monte Carlo and Zero T DWE-studies. We use Υ_4 to compare with the SI – SII transition.

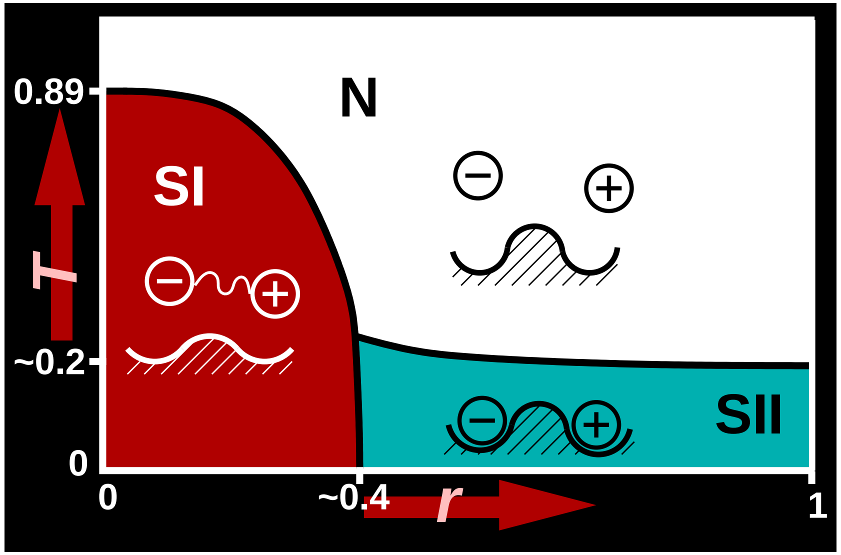


Gauge Glass (maximally disordered limit of the Random Gauge *XY* model)


0
−5
−10
−15
−20
0.0
0.1
0.2
0.3
0.4
T
L = 4
L = 6
L = 8
Y4
0
−2
−4
−6
−8
−10
−12

Random Gauge *XY* Model

- There exists a low-T superconducting phase for all values of r.
- In the large-r phase ergodicity is broken.



The *XY* Spin Glass Model

- For almost all s there is no low-T superconducting phase.
- There is a possibility of a chiral phase at low temperatures.

The Random Pinning Model

- There is no low-T superconducting phase.