Congestion and centrality in complex networks

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Motivation

- Many studies have used graph centrality measures to assess the load on vertices in communication networks
- Where congestion arises depends both on the network and the dynamical system.
- If the system is congestion sensitive the centrality measures may fail to capture the load.
- We test how the betweenness centrality correlates with dynamical centrality measures.

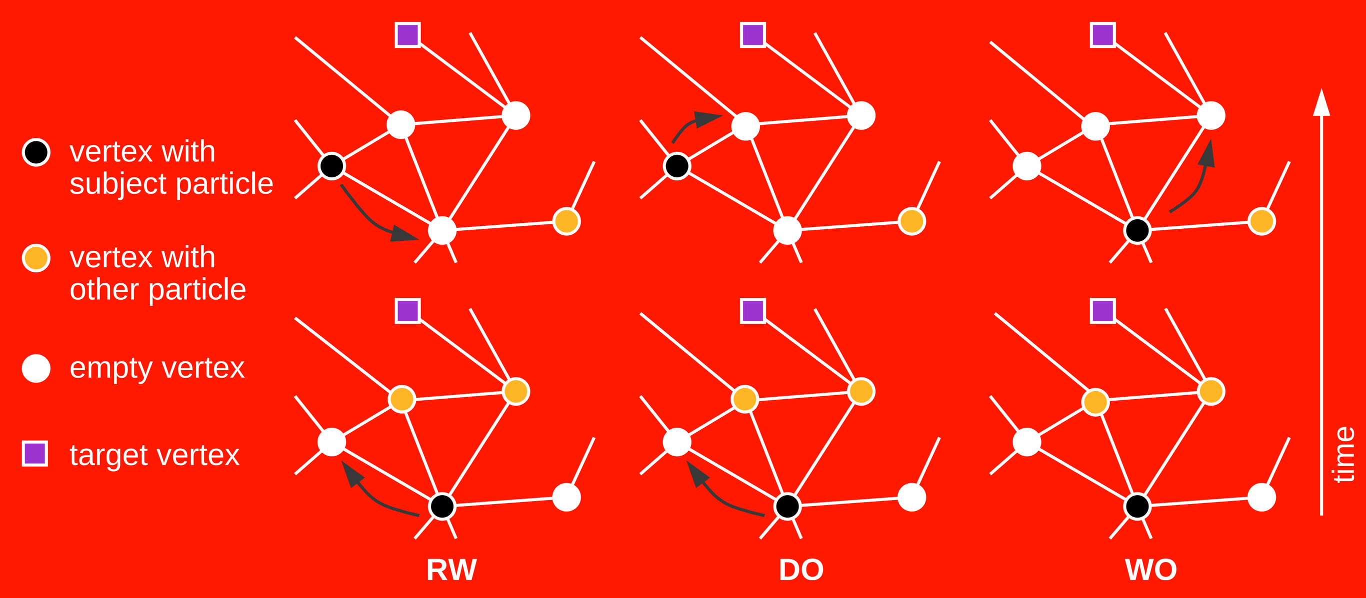
Model

- To model a congestion sensitive communication system on a graph $G = (V, E)$ we consider fN particles (agents) that each are characterized by a start and a target vertex.
- Maximum one particle is allowed at each vertex.
- The particles are updated sequentially. They move to a neighboring vertex according to their strategy:

Strategies

Random Walk (RW): A particle moves to a randomly selected, free, neighbor.

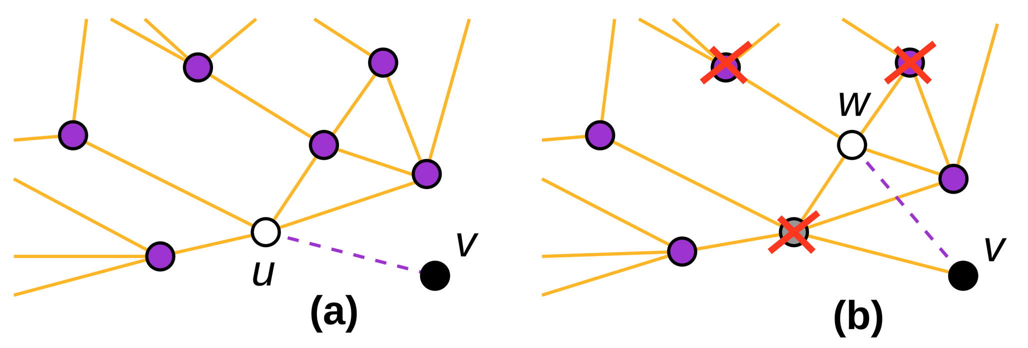
Detour Obstacle (DO): A particle chooses randomly among the vertices that lies closest to the target. If all such are occupied it moves to a vertex equidistant to the target. If no such are free either it moves away from the target.



Wait at Obstacle (WO): Like DO but if no vertex closer to the target is free, then the particle waits.

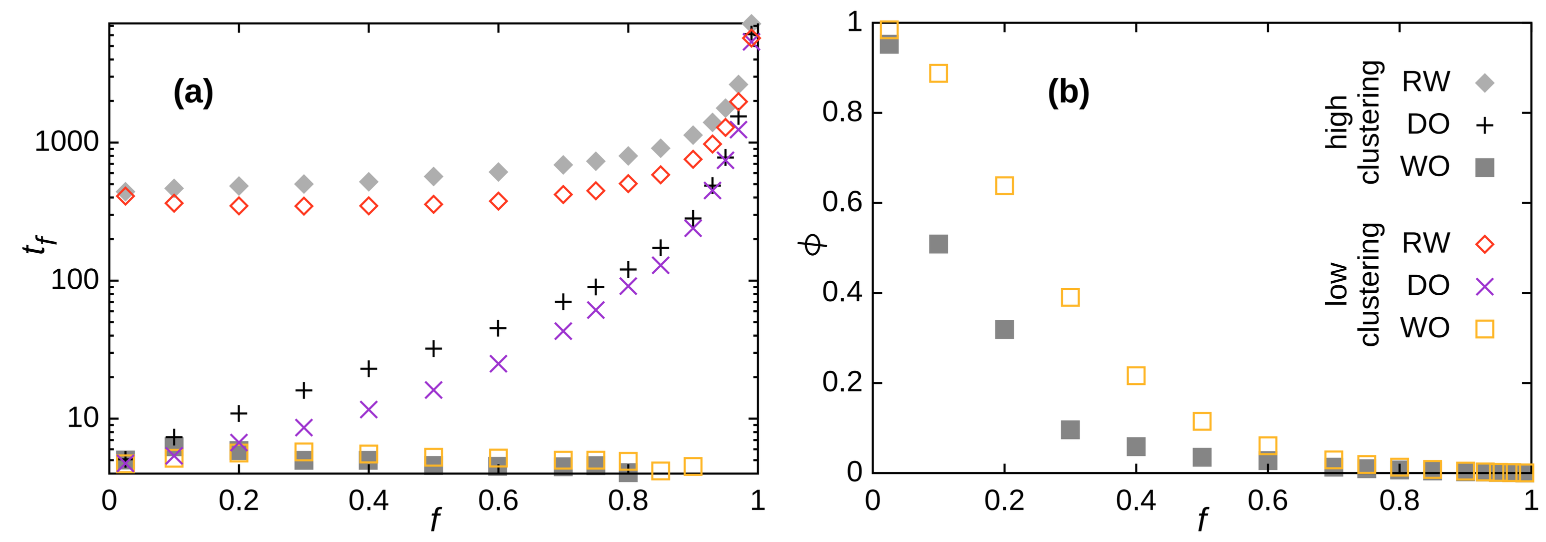
The networks

(a) In the preferential attachment step for the newly added vertex v (denoted as filled black circle), the white vertex u is chosen with the probability proportional to its degree (the dotted line represents the new edge). **(b)** In the triangle formation step an additional edge (dotted line) is added to a randomly selected vertex w in the neighborhood Γ_u of the vertex u chosen in the previous preferential attachment step in (a). The crossed out vertices are not allowed since they are not in Γ_u. Without the triangle formation step (i.e. if $m_t = 0$), the clustered scale-free model reduces to the original BA model of scale-free networks.



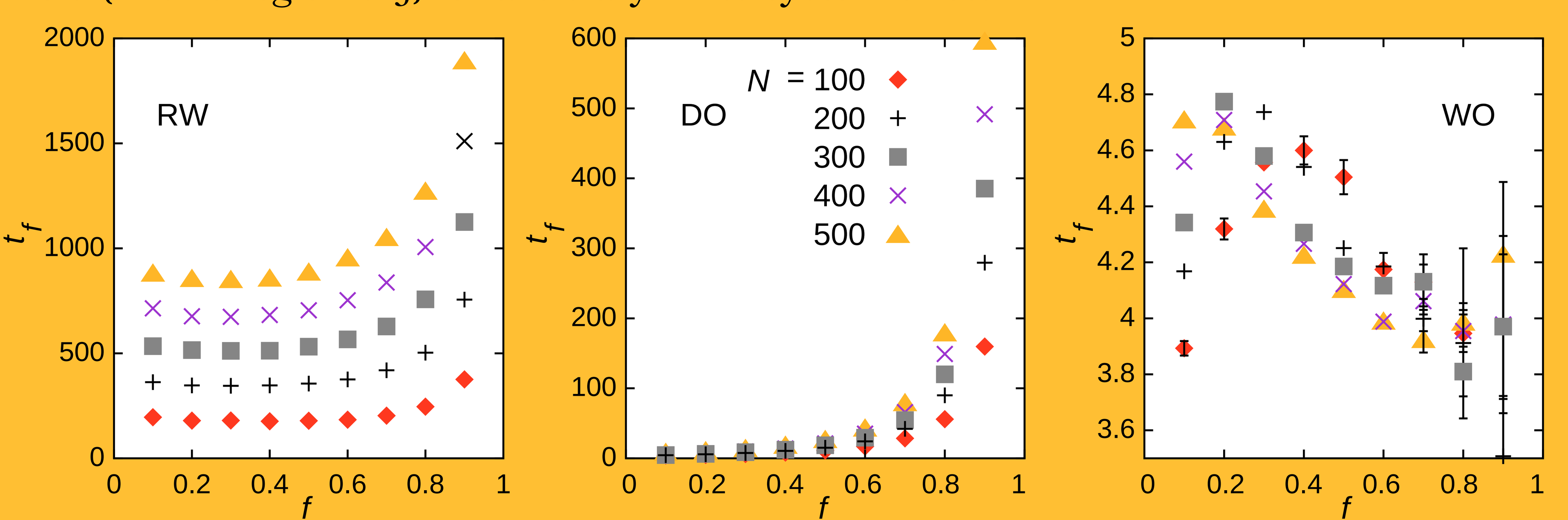
Speed of the dynamics

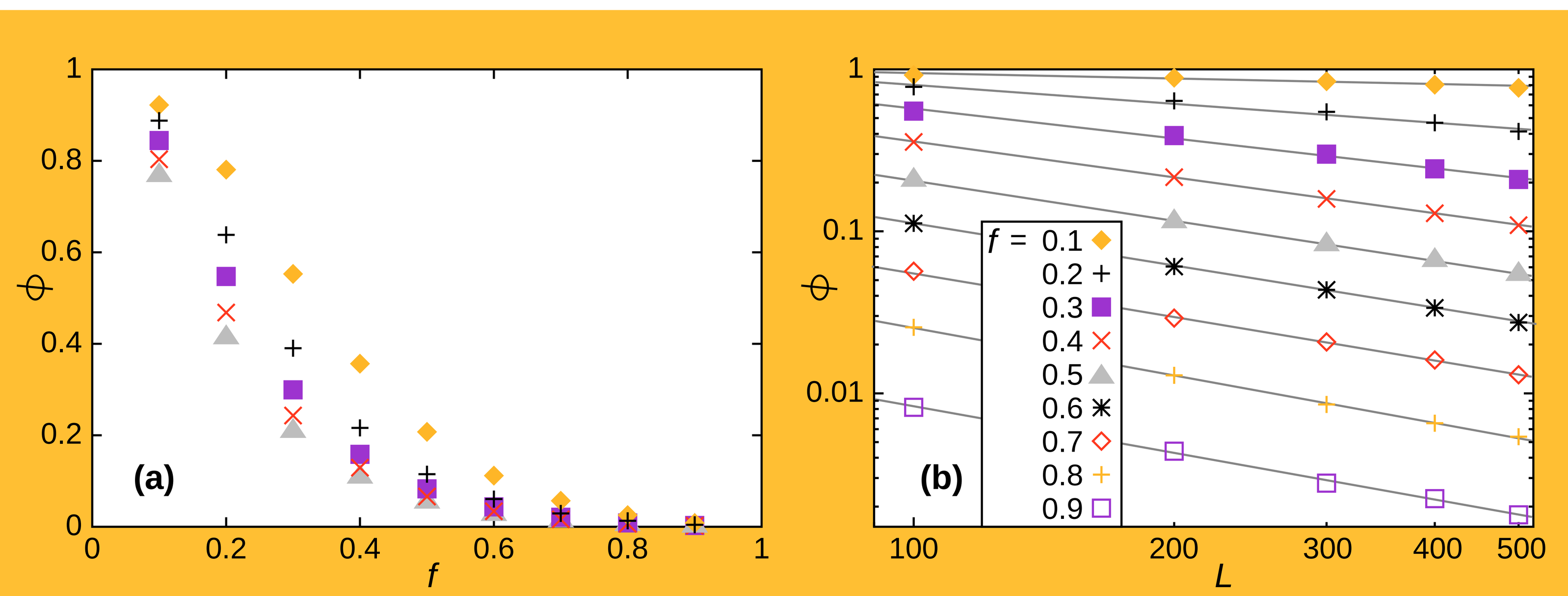
The traffic-density dependence of speed of the dynamics: Average finding t_f time for pairs at distance four as a function of particle density for the three different types of dynamics, and both low ($m_t = 0$ giving $C = 0.056$ for thepresent values of m, m_0 and N) and high clustering ($m_t = 2.8$ giving $C = 0.24$). (b) shows the fraction of pairs that reaches their targets ϕ as a function of f in the WO dynamics. For the RW and DO updating rules ϕ is strictly unity. The other model parameters are $N = 200$ and $m = m_0 = 3$.



The effect of traffic density

Finding time of vertex-pairs at distance four for different system sizes. The system parameters are $m_t = 0$, $m = m_0 = 3$. The lack of emerging singularities for $0.1 < f < 0.9$ implies that qualitative conclusions from moderate system sizes will hold (in this region of f) for arbitrary finite system sizes.

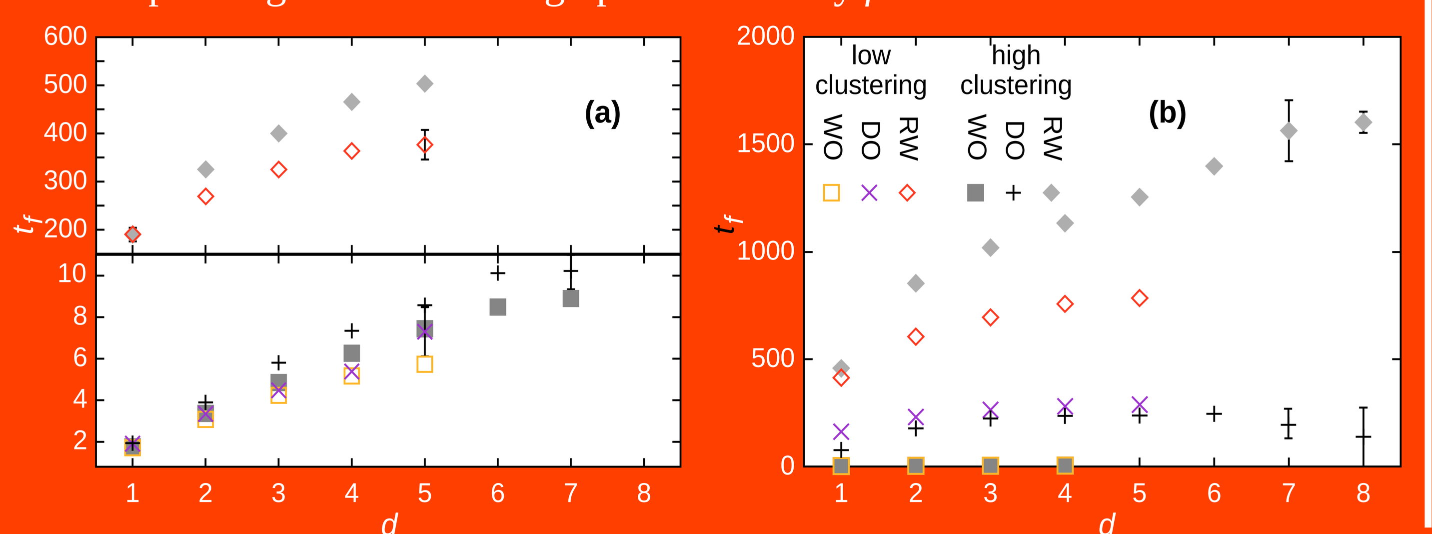




Finite size scaling of the WO dynamic's finding probability ϕ. **(a)** shows ϕ as a function of f while **(b)** displays ϕ as a function of system size. The lines are curve-fits to a power-law form. Just as the RW and DO dynamics finding times goes to infinity (as seen above), ϕ goes to zero, which shows that all three dynamics are (from a statistical mechanics point of view) in a congested state.

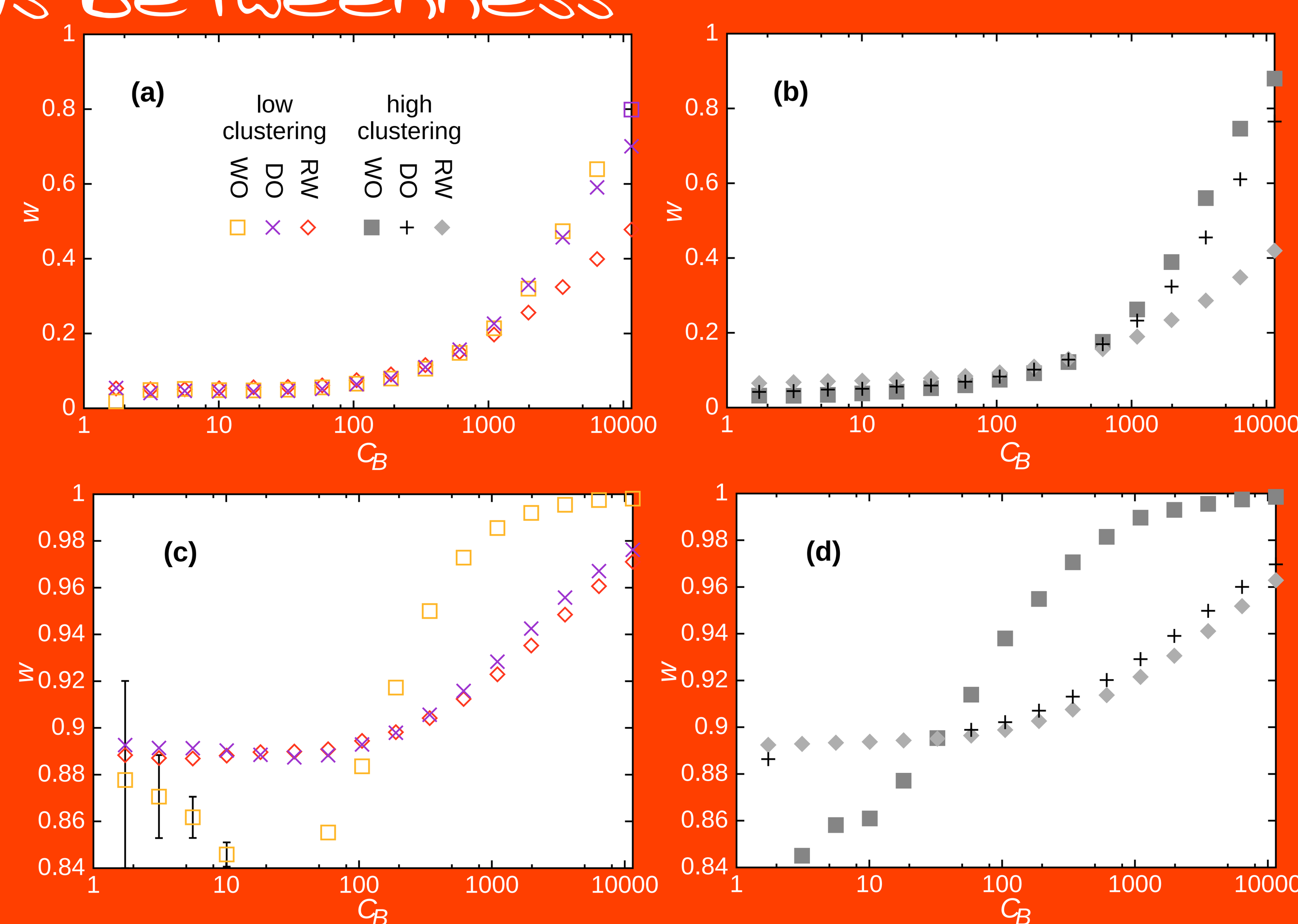
Finding time vs distance to target

Finding time as a function of geodesic distance to the target. **(a)** shows the situation for low particle density $f = 0.1$. **(b)** shows the corresponding situation for high particle density $f = 0.9$.



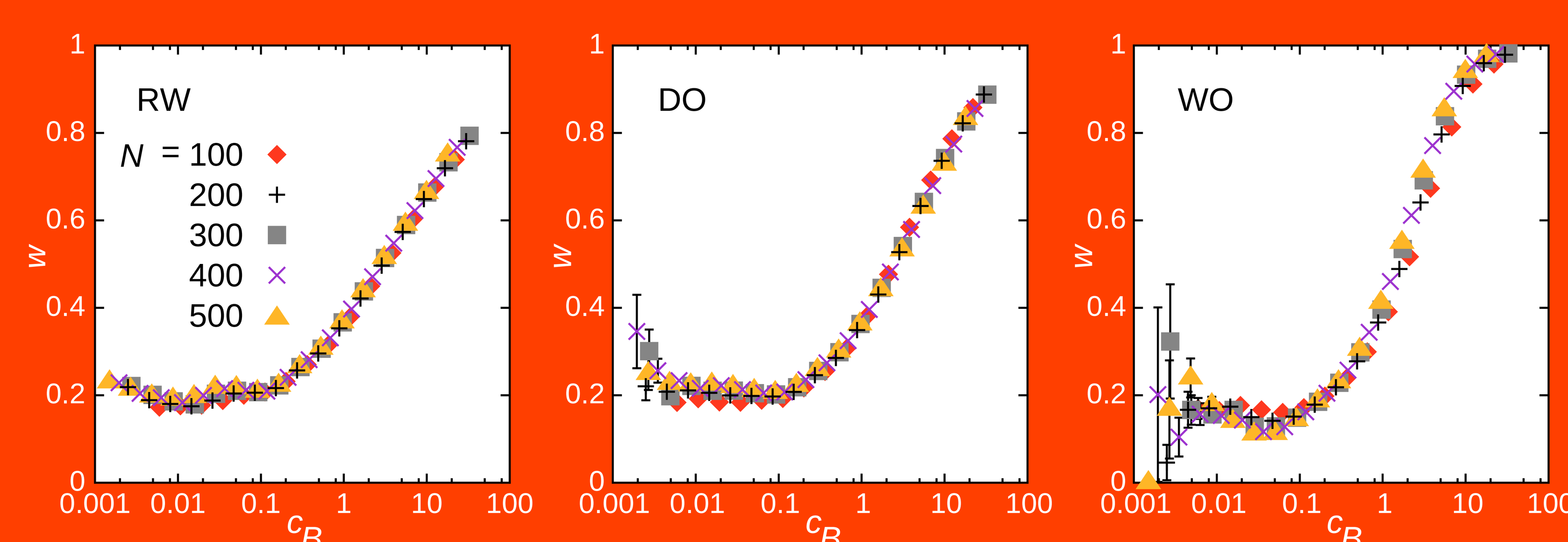
Occupation ration vs betweenness

Occupation time as a function of betweenness. (a) and (b) shows the three dynamics at the low particle density $f = 0.1$; (c) and (d) shows the corresponding situation for a high particle density $f = 0.9$. (a) and (c) shows networks of low clustering $m_t = 0$ (giving $C = 0.056$) while (b) and (d) have high clustering $m_t = 2.8$ (giving $C = 0.24$). The other model parameters are $N = 200$ and $m = m_0 = 3$.



Occupation ration vs rescaled betweenness

Occupation ratio as a function of normalized betweenness for different N. The other model parameters are $f = 0.3$, $m_t = 0$, and $m = m_0 = 3$. The curves overlap as N grows indicating that the qualitative picture is the same for arbitrary large sizes.



Conclusions

- In networks with low traffic density the WO strategy is the fastest, otherwise DO is fastest.
- Betweenness is not proportional to the occupation ratio (the dynamical congestion measure).
- The reason betweenness fails as a load measure is that neighbors of central vertices get a heavy load as well.

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