



# MODELING THE COEVOLUTION OF NETWORKS AND OPINIONS

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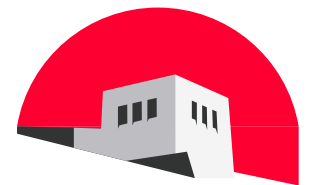
*e-print physics/0603023*

with

**Mark Newman**

University of Michigan, Ann Arbor, USA

<http://www.cs.unm.edu/~holme/>

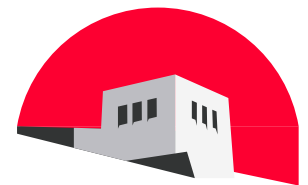


# the idea

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- ★ Opinions spread over social networks.

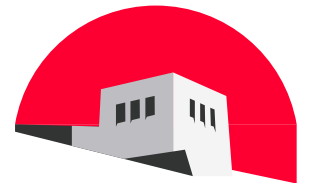


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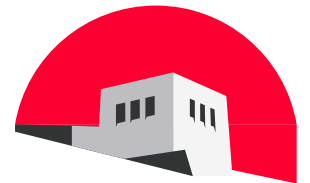
- ★ Opinions spread over social networks.
- ★ People with the same opinion are likely to become acquainted.



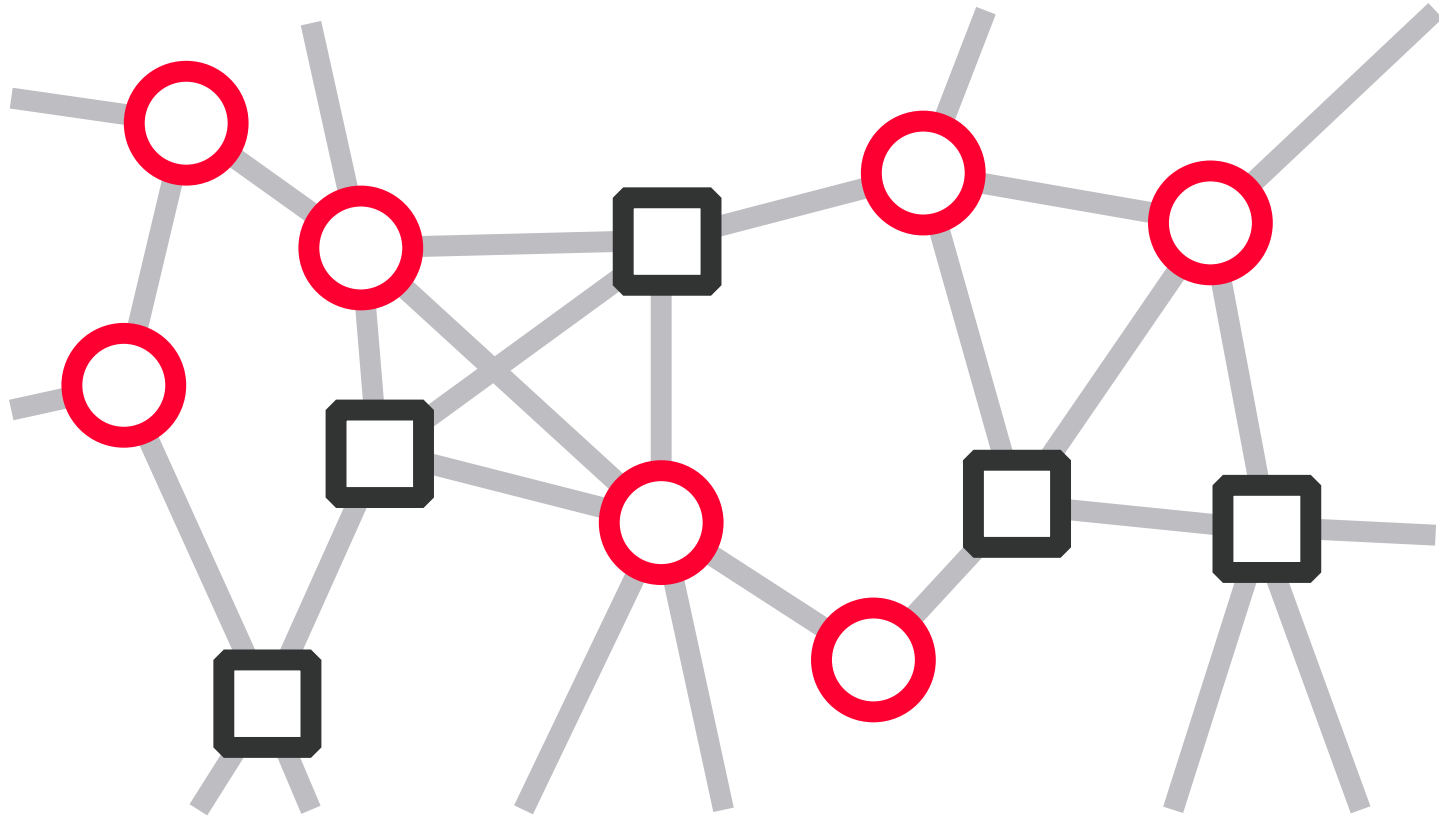
# the idea



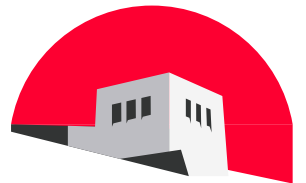
- ★ Opinions spread over social networks.
- ★ People with the same opinion are likely to become acquainted.
- ★ We try to combine these points into a simple model of simultaneous opinion spreading and network evolution.



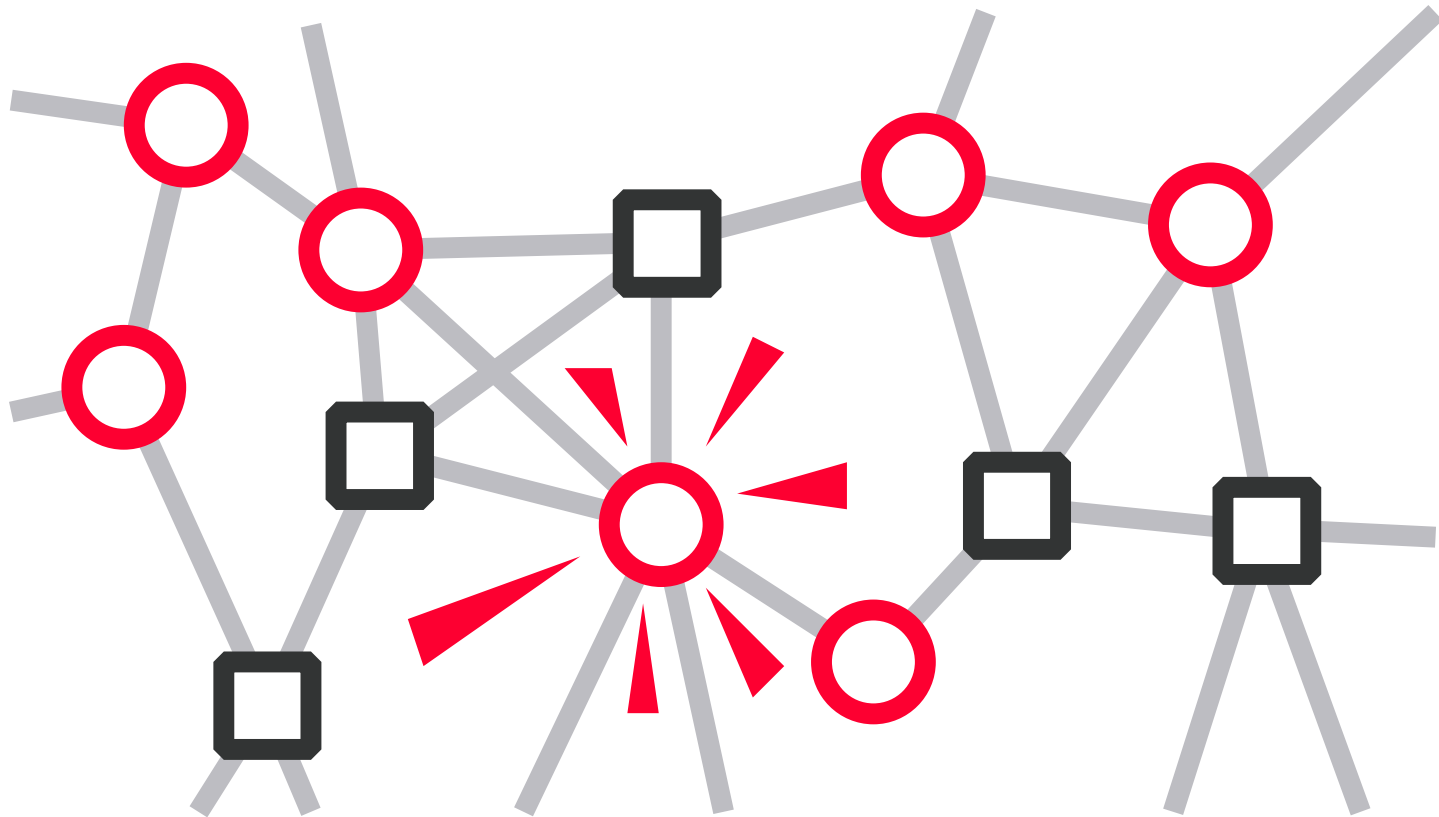
# the voter model



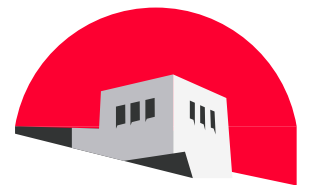
Clifford & Sudbury, *Biometrika* **60**, 581 (1973).  
Holley & Liggett, *Ann. Probab.* **3**, 643 (1975).



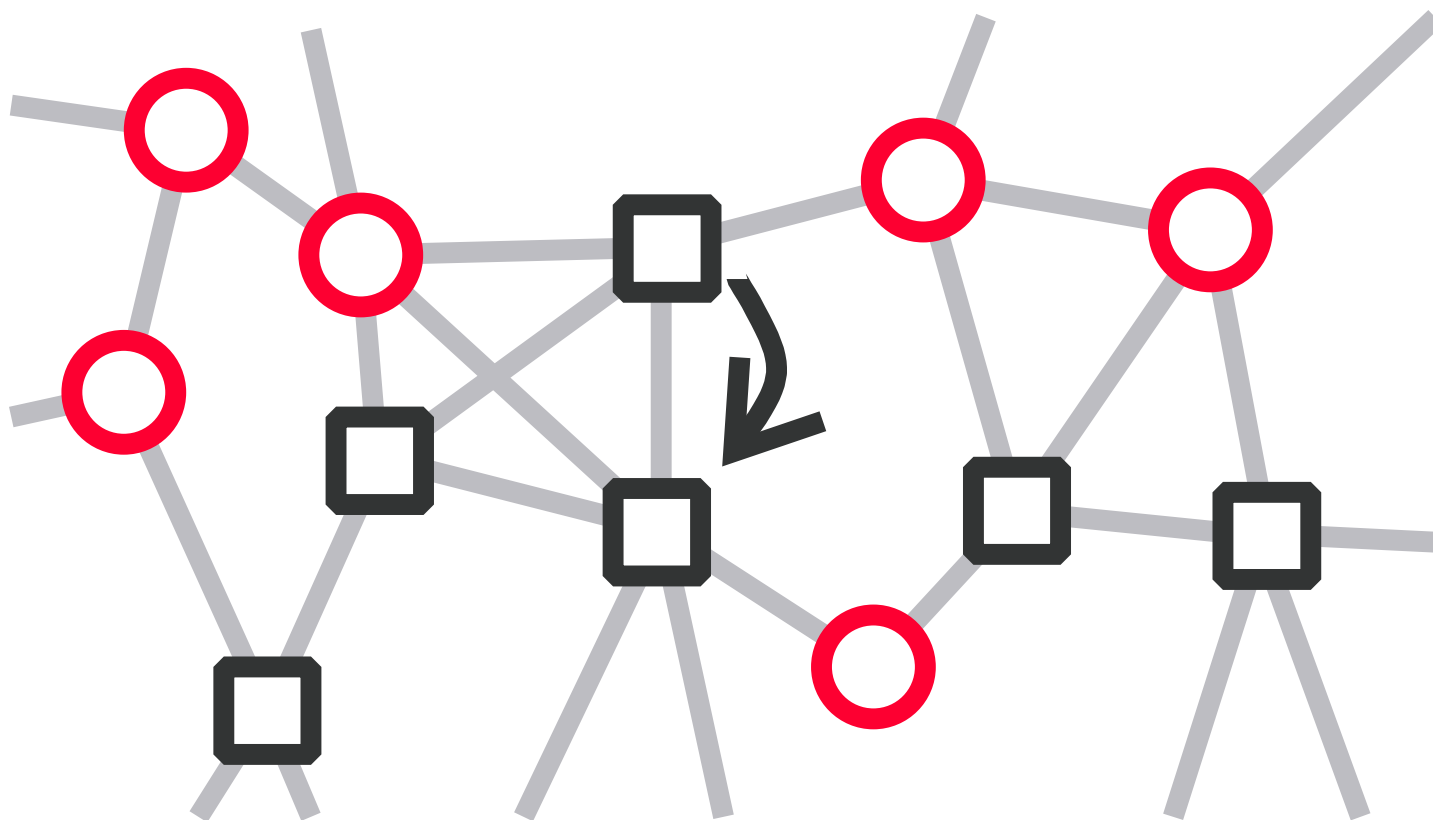
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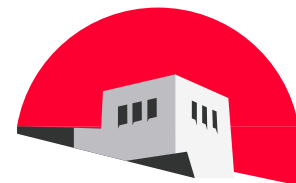
choose one vertex randomly



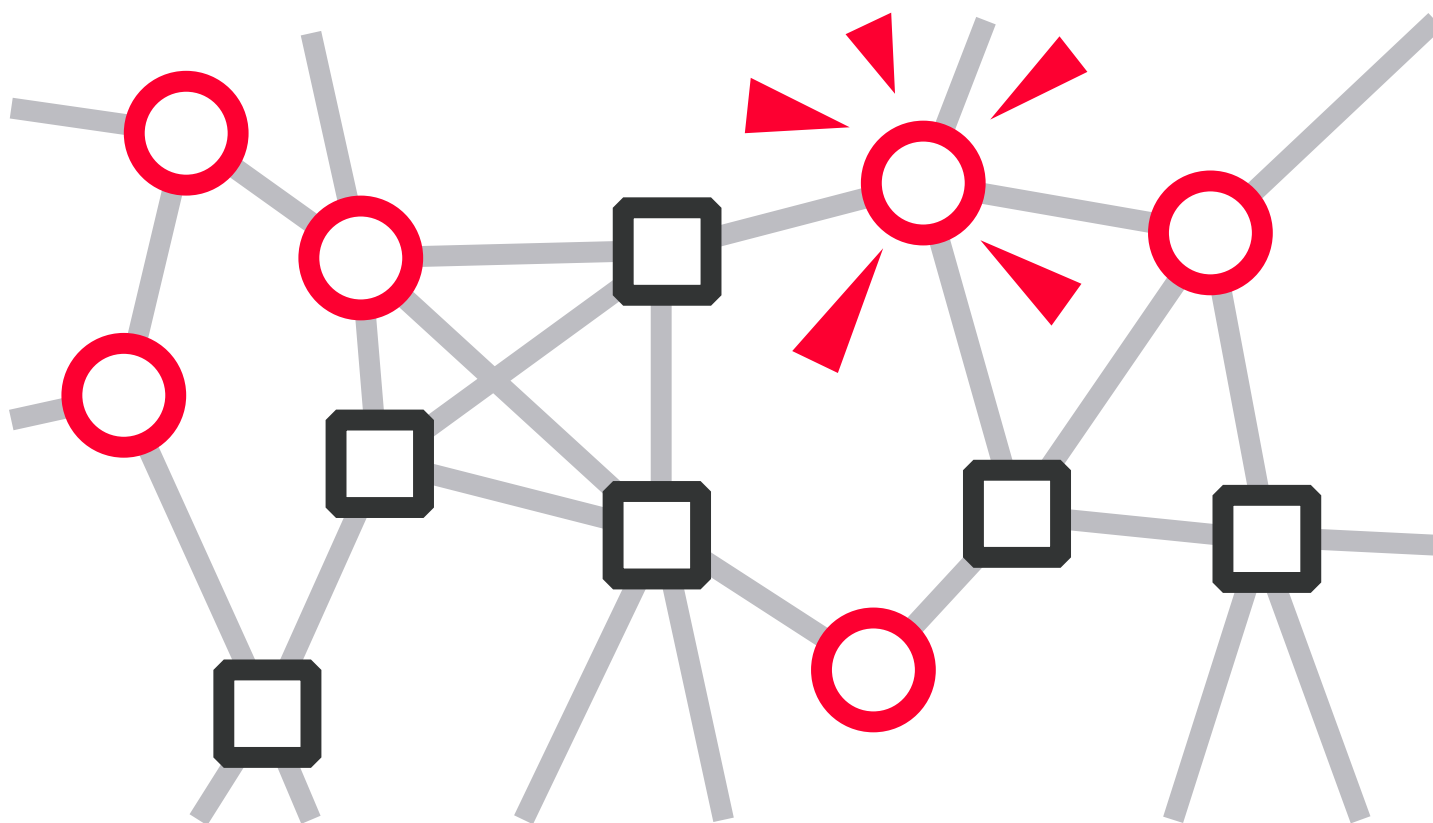
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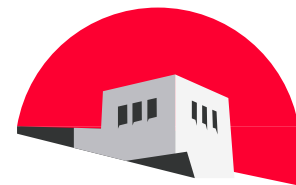
copy the opinion of a random neighbor



# the voter model

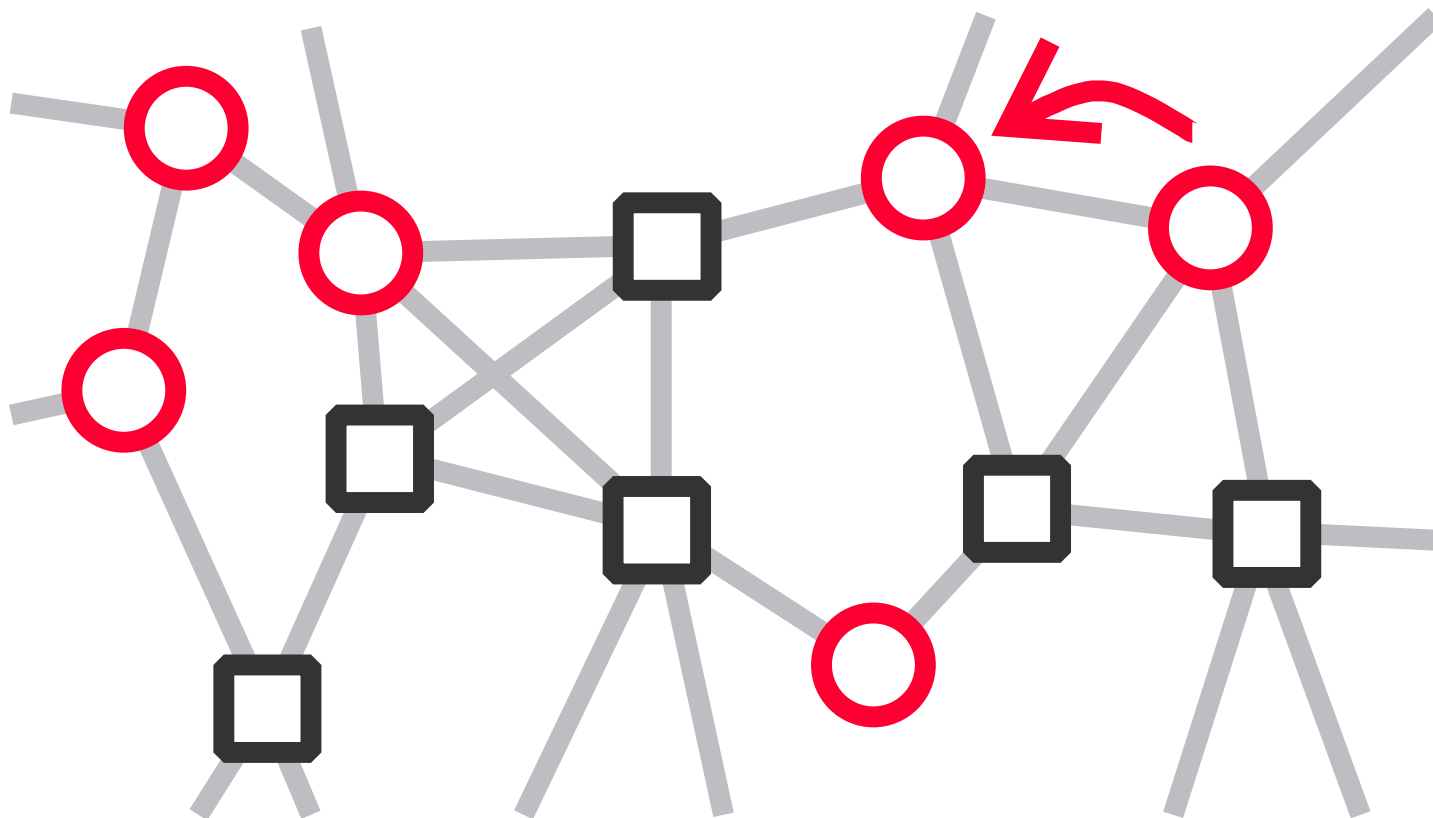


and so on . . .

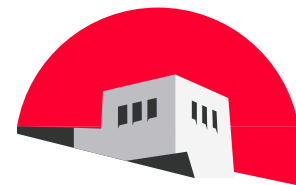




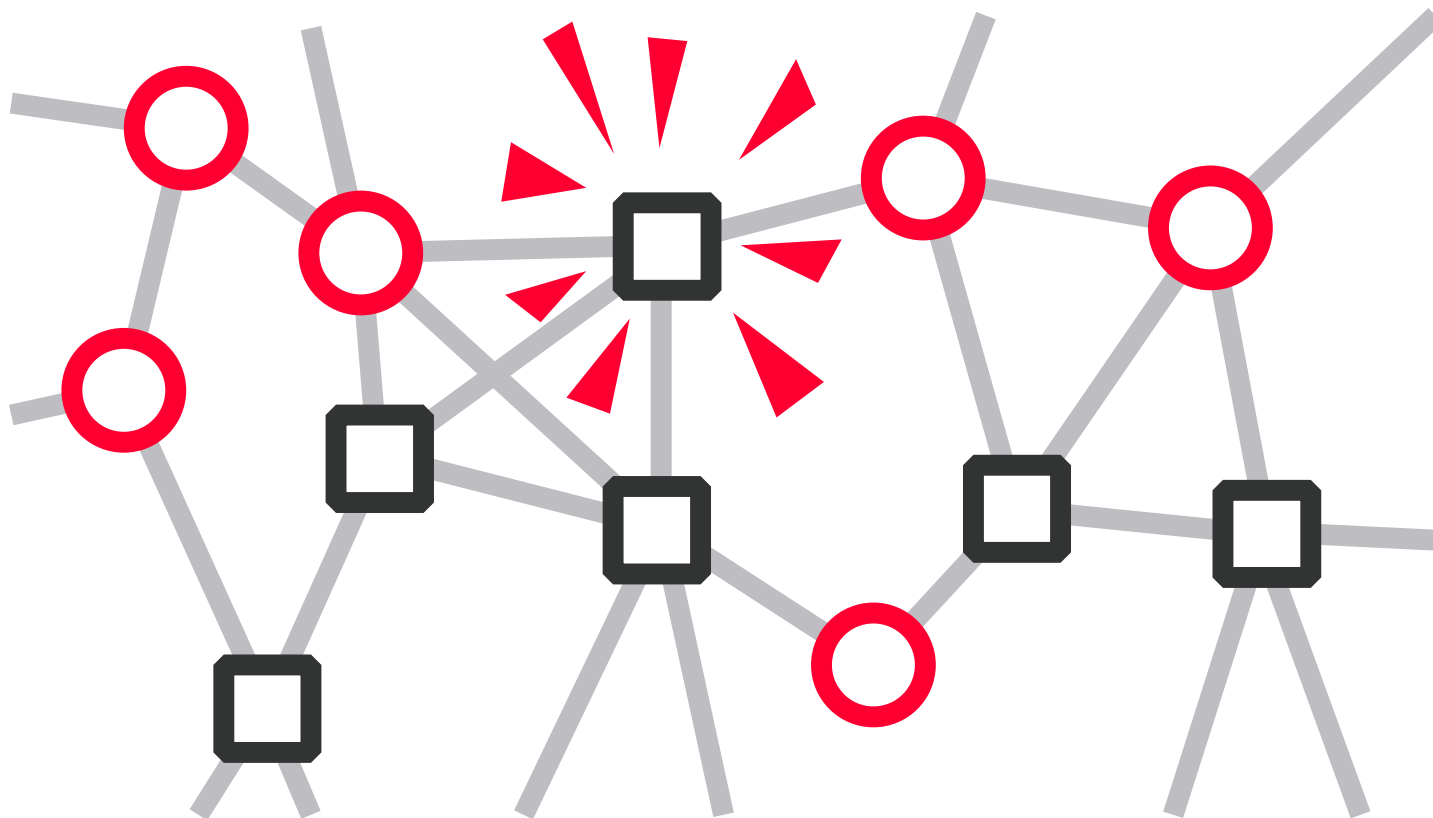
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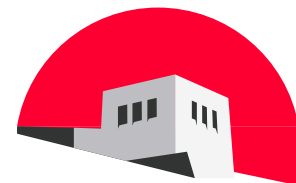
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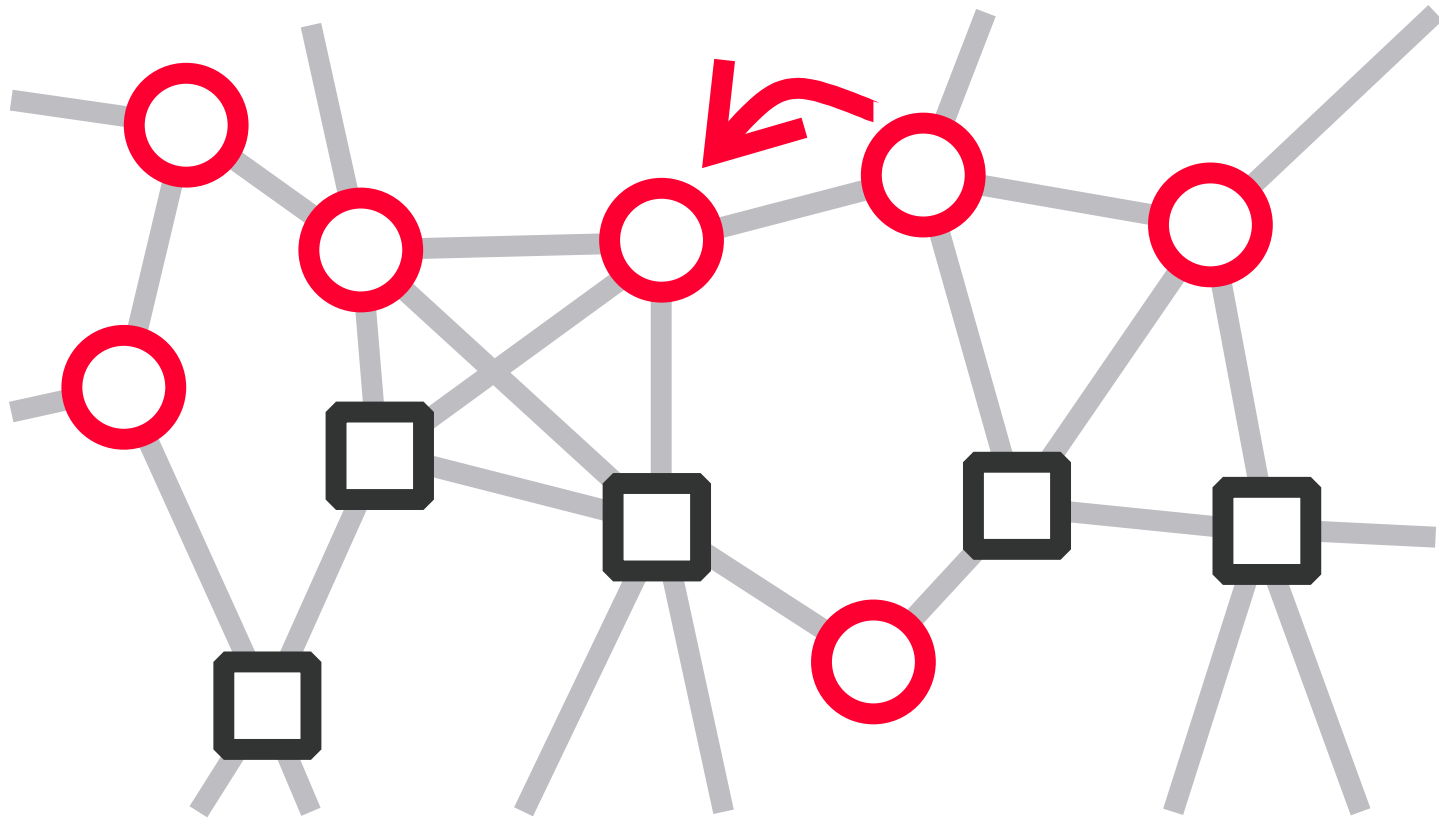
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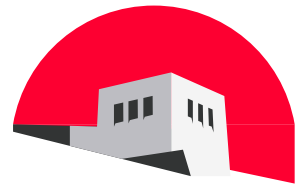
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# the voter model



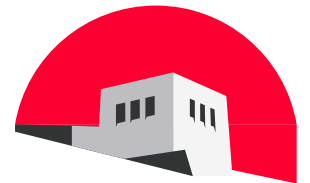
and so on . . .



# acquaintance dynamics



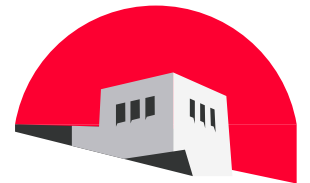
- ★ People of similar interests are likely to get acquainted. e.g.:  
McPherson *et al.*, *Ann. Rev. Sociol.* **27**, 415 (2001).



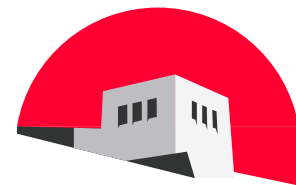
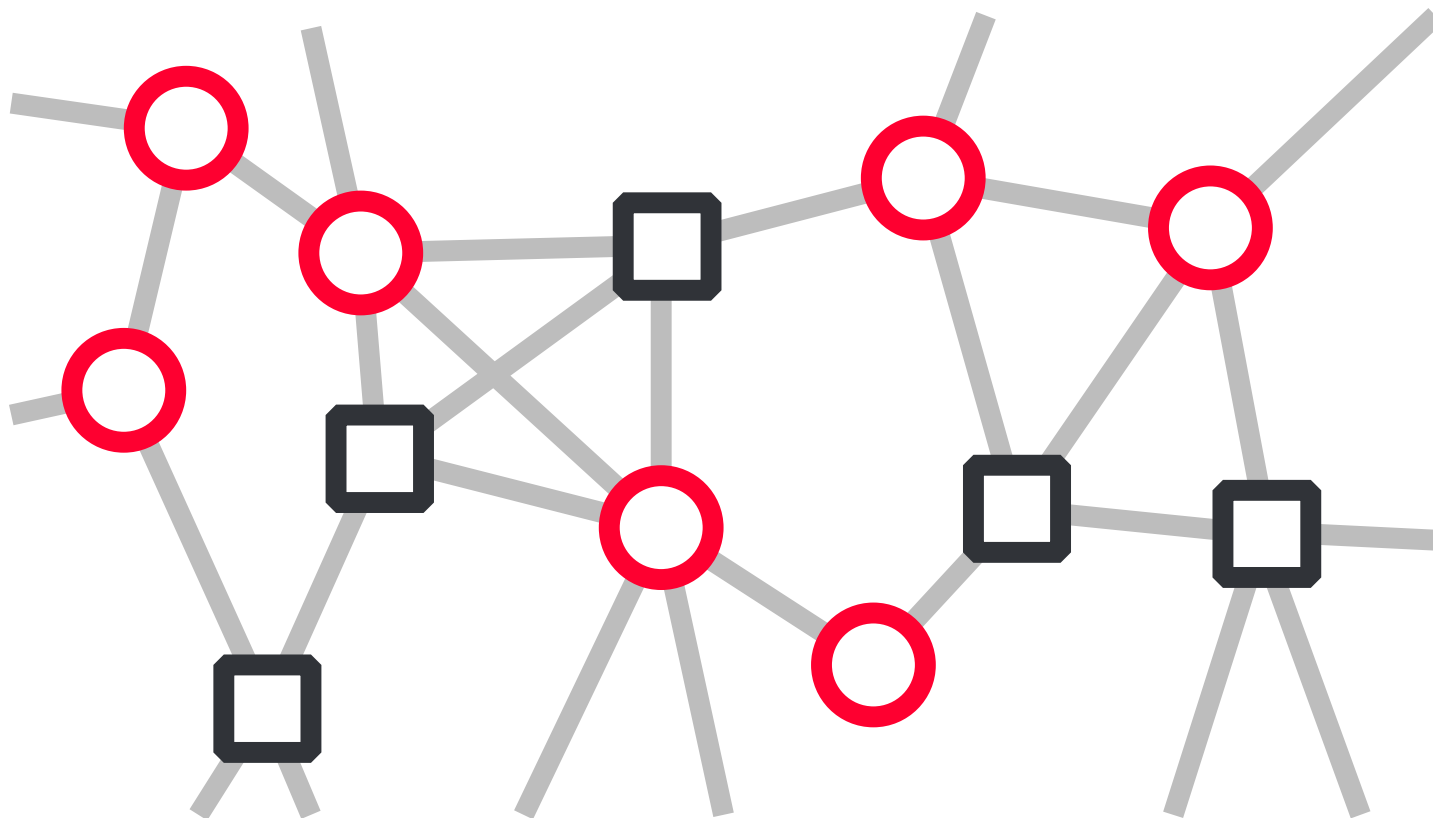
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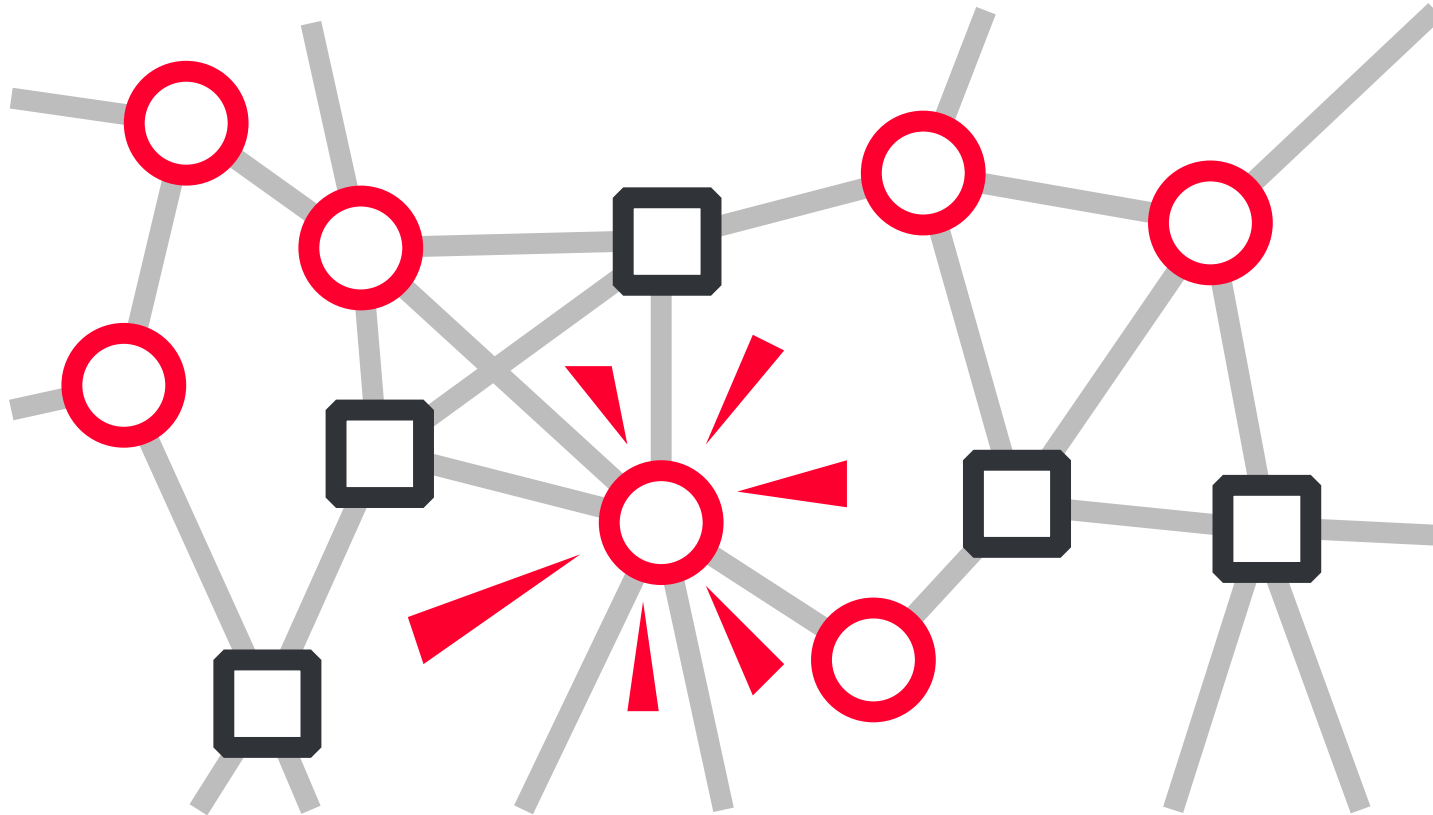
- ★ People of similar interests are likely to get acquainted. e.g.: McPherson *et al.*, *Ann. Rev. Sociol.* **27**, 415 (2001).
- ★ The number of edges is constant.



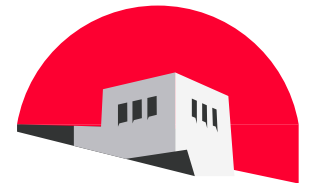
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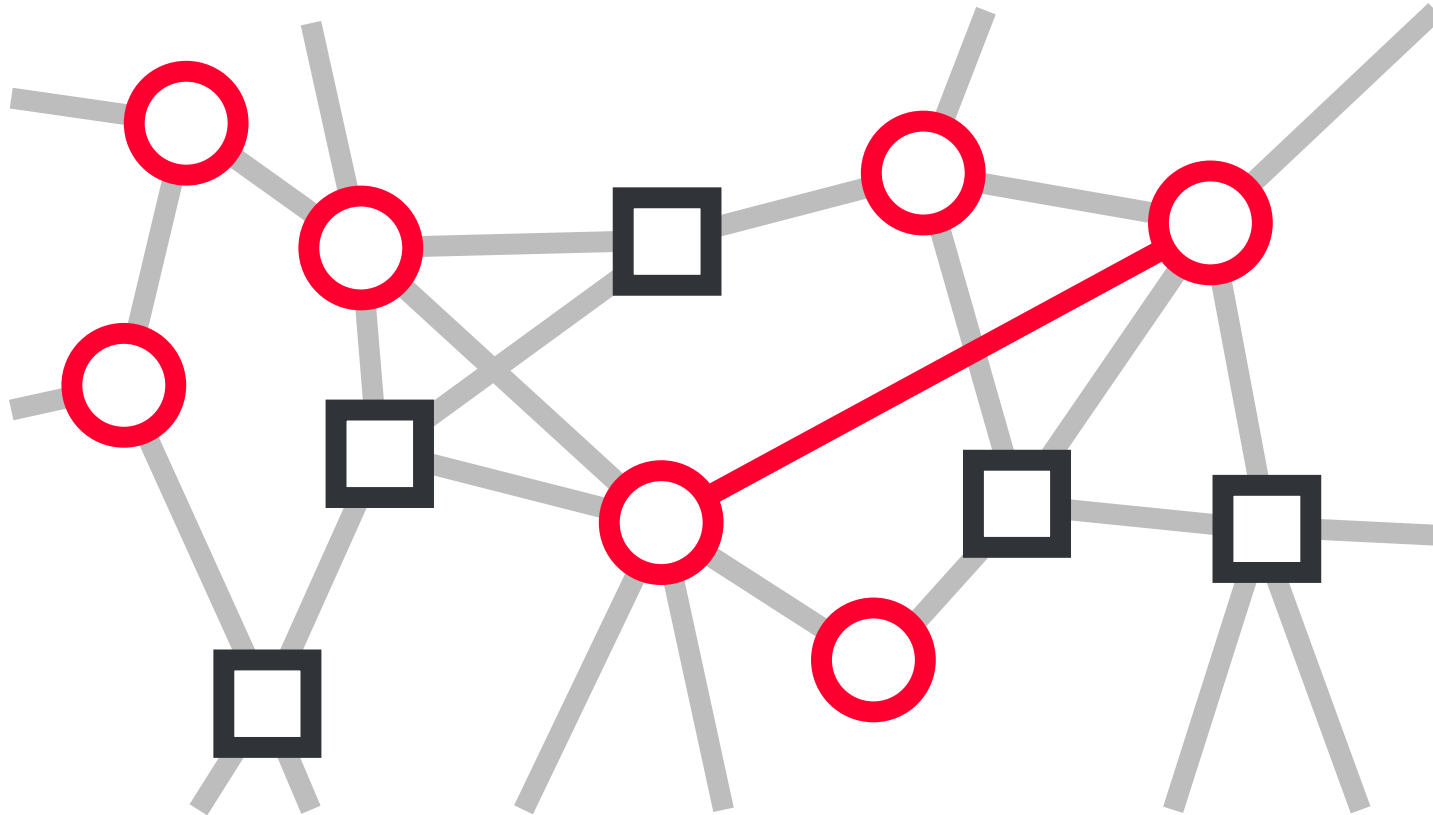
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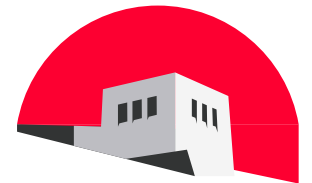
choose one vertex randomly



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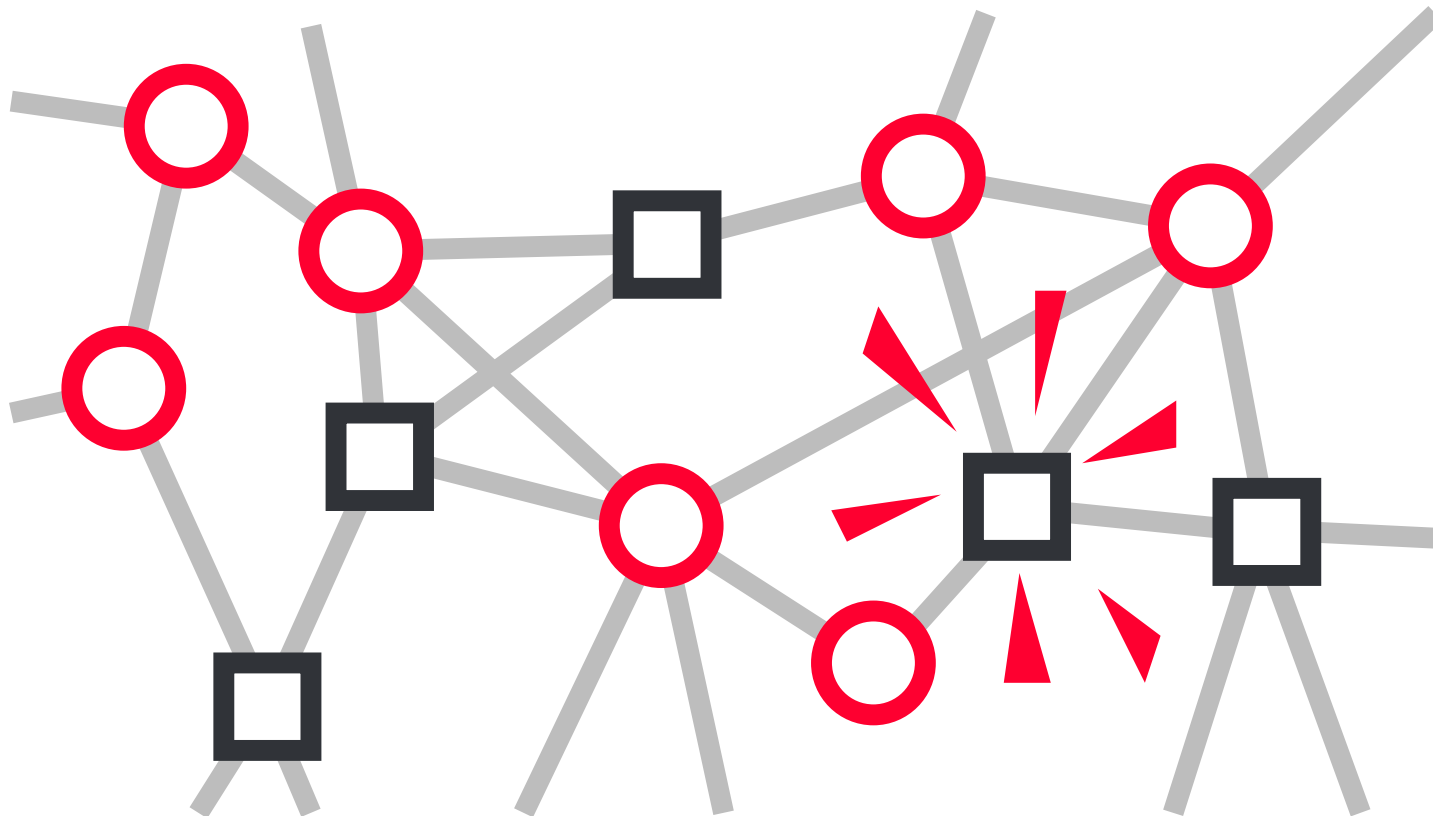


rewire an edge to a vertex with the same opinion

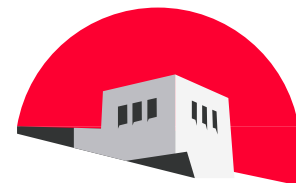




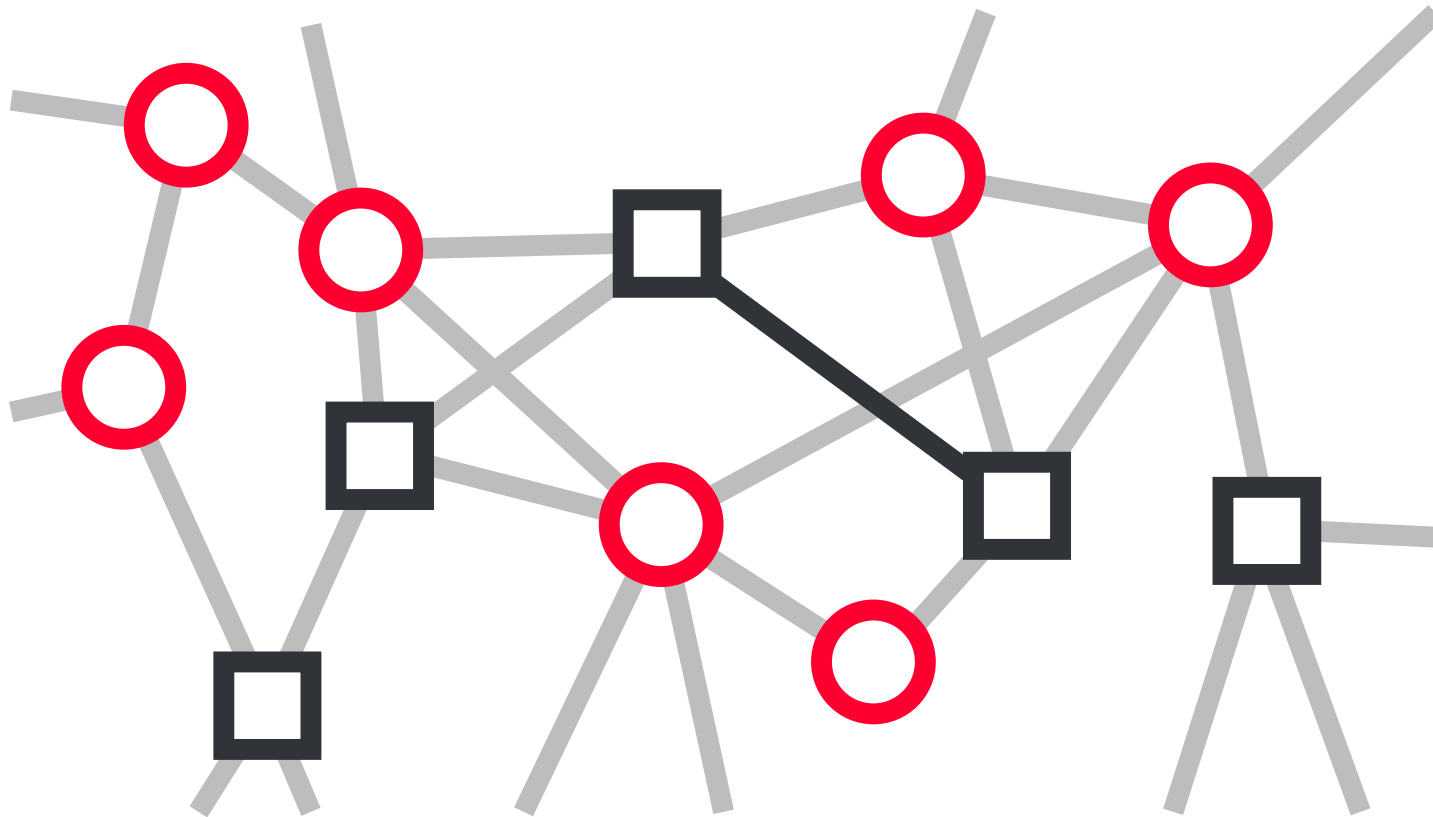
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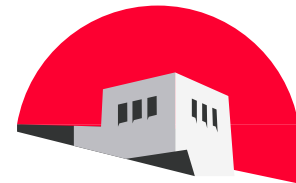
and so on . . .



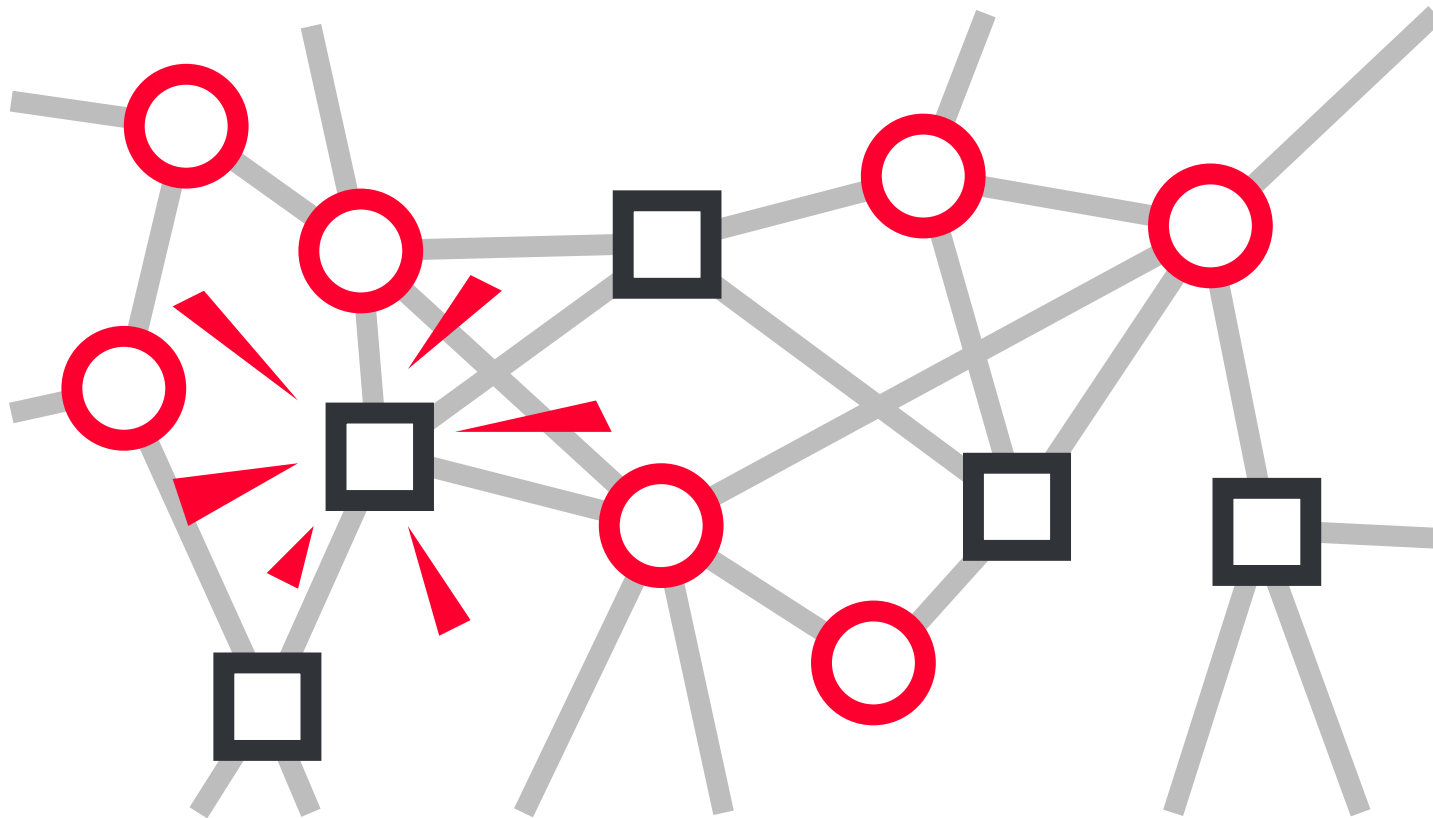
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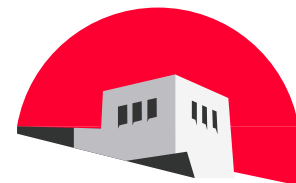
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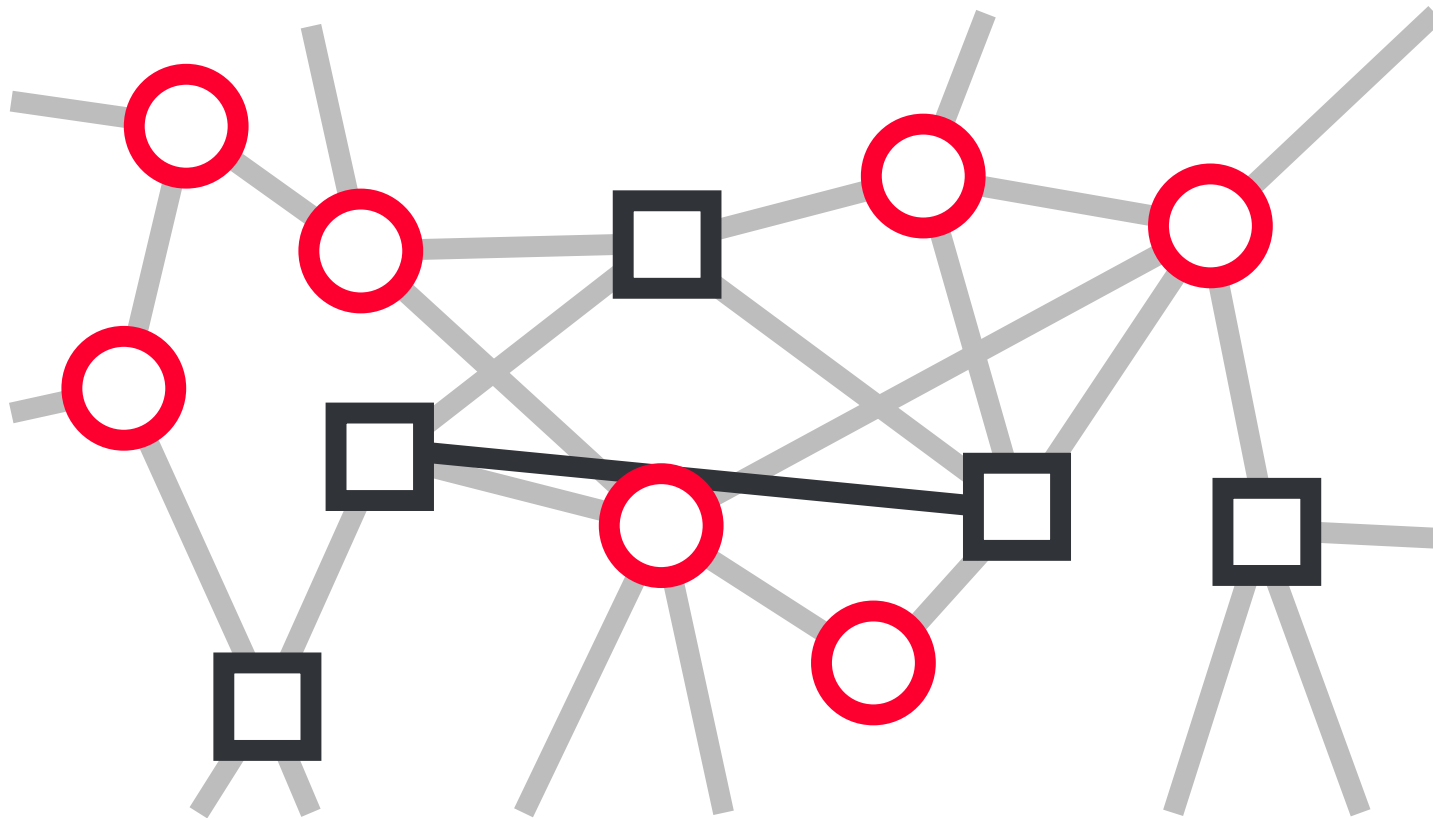
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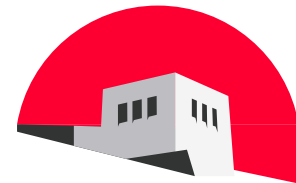
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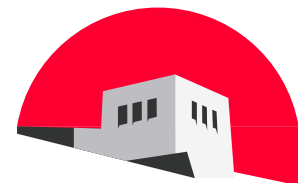
and so on . . .



# our model



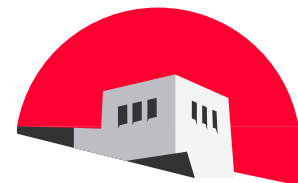
1. Start with a random network of  $N$  vertices  $M = \bar{k}N/2$  edges and  $G = N/\gamma$  randomly assigned opinions.



# our model



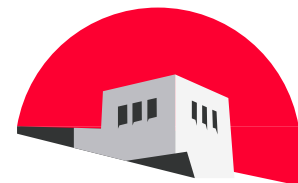
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2. Pick a vertex  $i$  at random.



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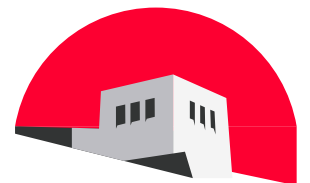
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3. With a probability  $\phi$  make a acquaintance formation step from  $i$  . . .



# our model



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4. . . . otherwise make a voter model step from  $i$ .

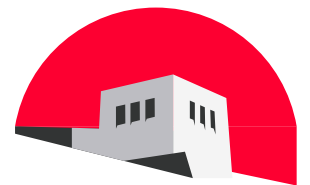




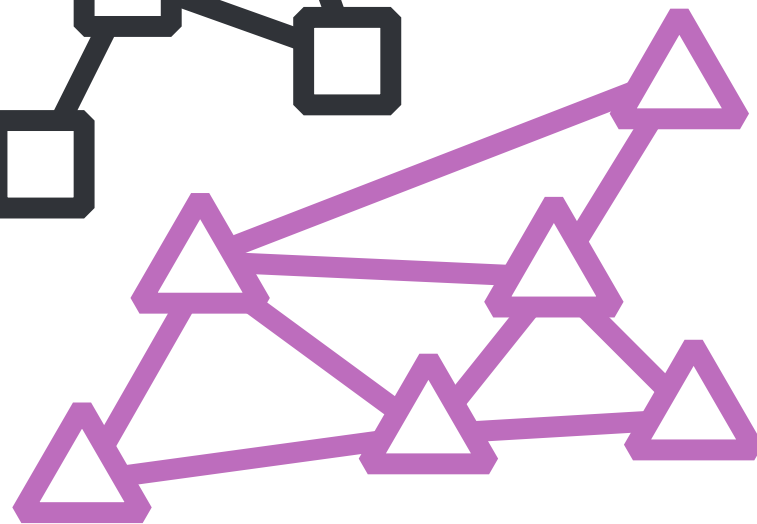
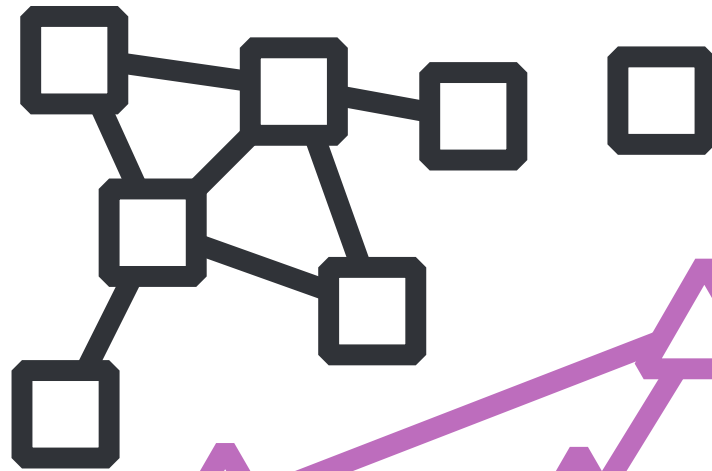
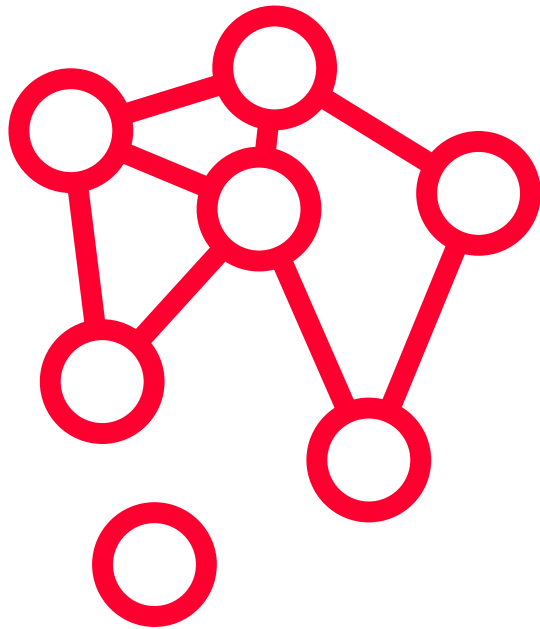
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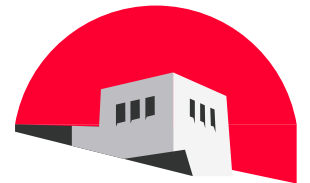
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2. Pick a vertex  $i$  at random.
3. With a probability  $\phi$  make a acquaintance formation step from  $i$  . . .
4. . . . otherwise make a voter model step from  $i$ .
5. If there are edges leading between vertices of different opinions—iterate from step 2.



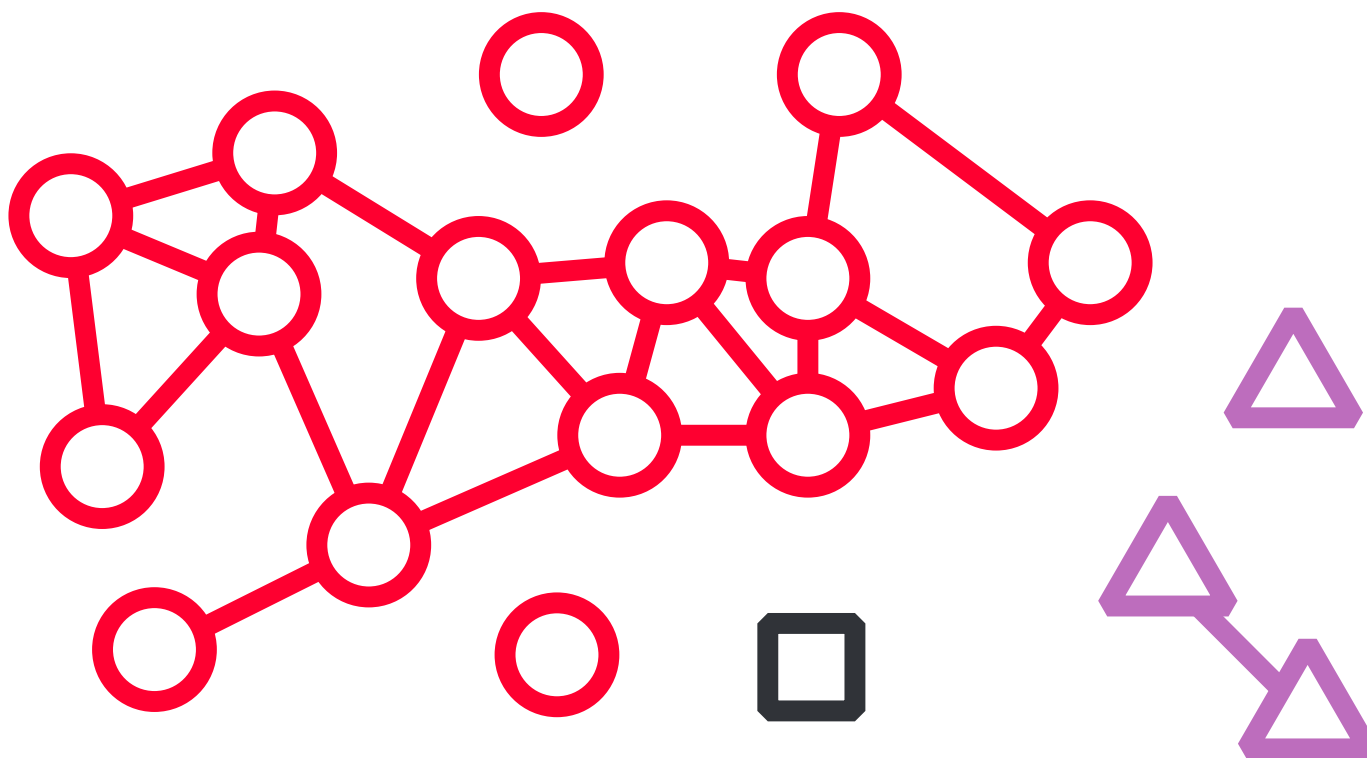
# phases



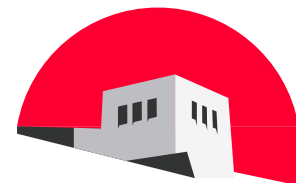
low  $\phi$ —clusters of similar sizes



# phases



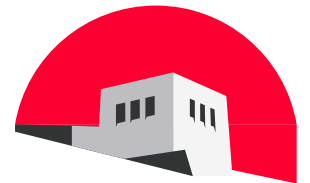
high  $\phi$  — one dominant cluster



# quantities we measure



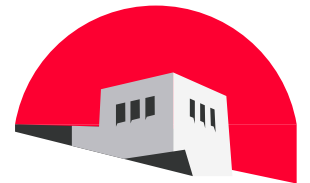
- ★ The relative size  $s = S/N$  of a cluster (of vertices with the same opinion).



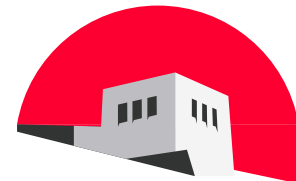
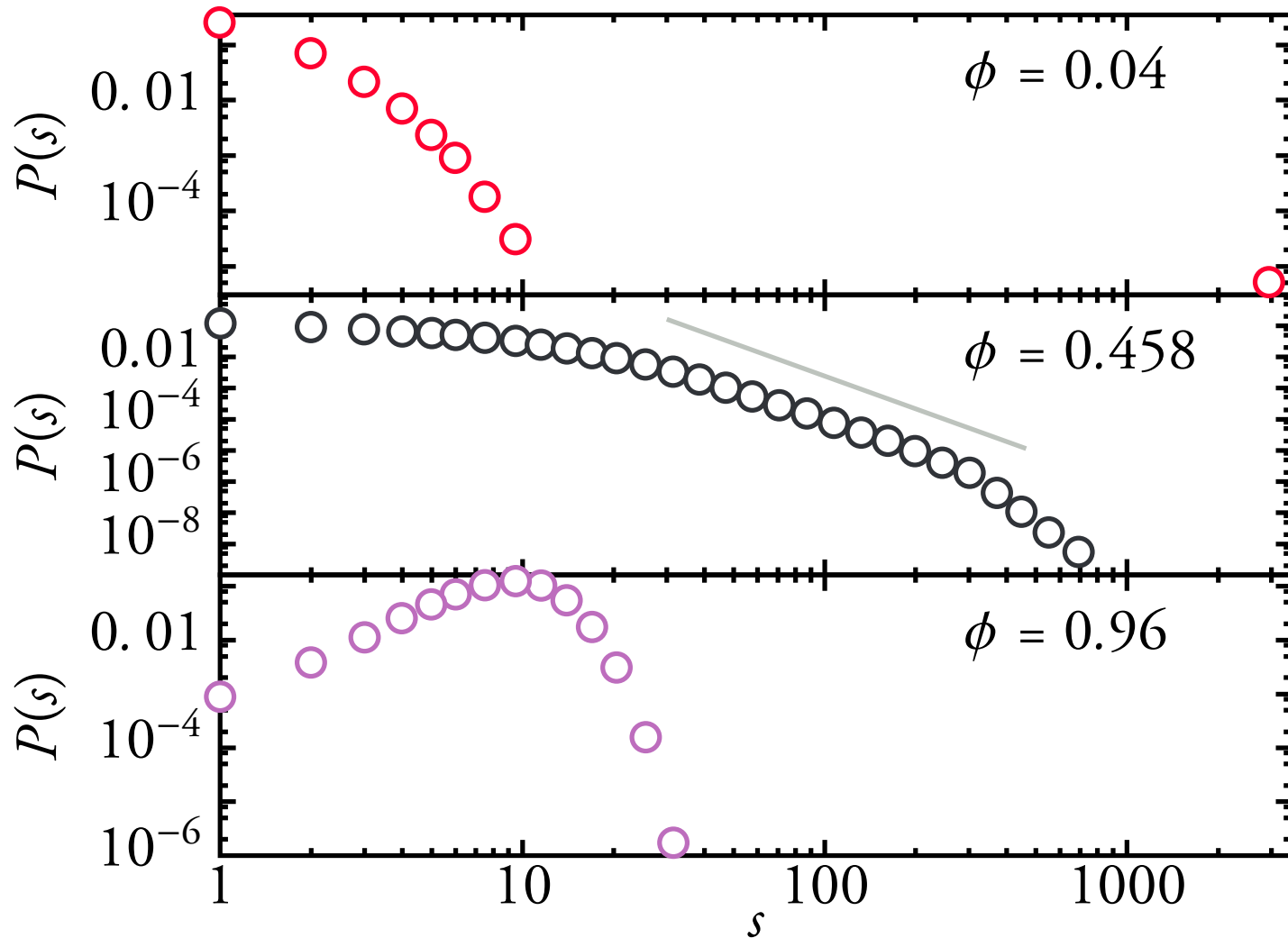
# quantities we measure



- ★ The relative size  $s = S/N$  of a cluster (of vertices with the same opinion).
- ★ The average time  $\tau$  to reach consensus.



# cluster size distribution

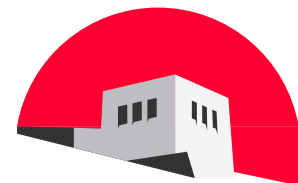


# finding the phase transition

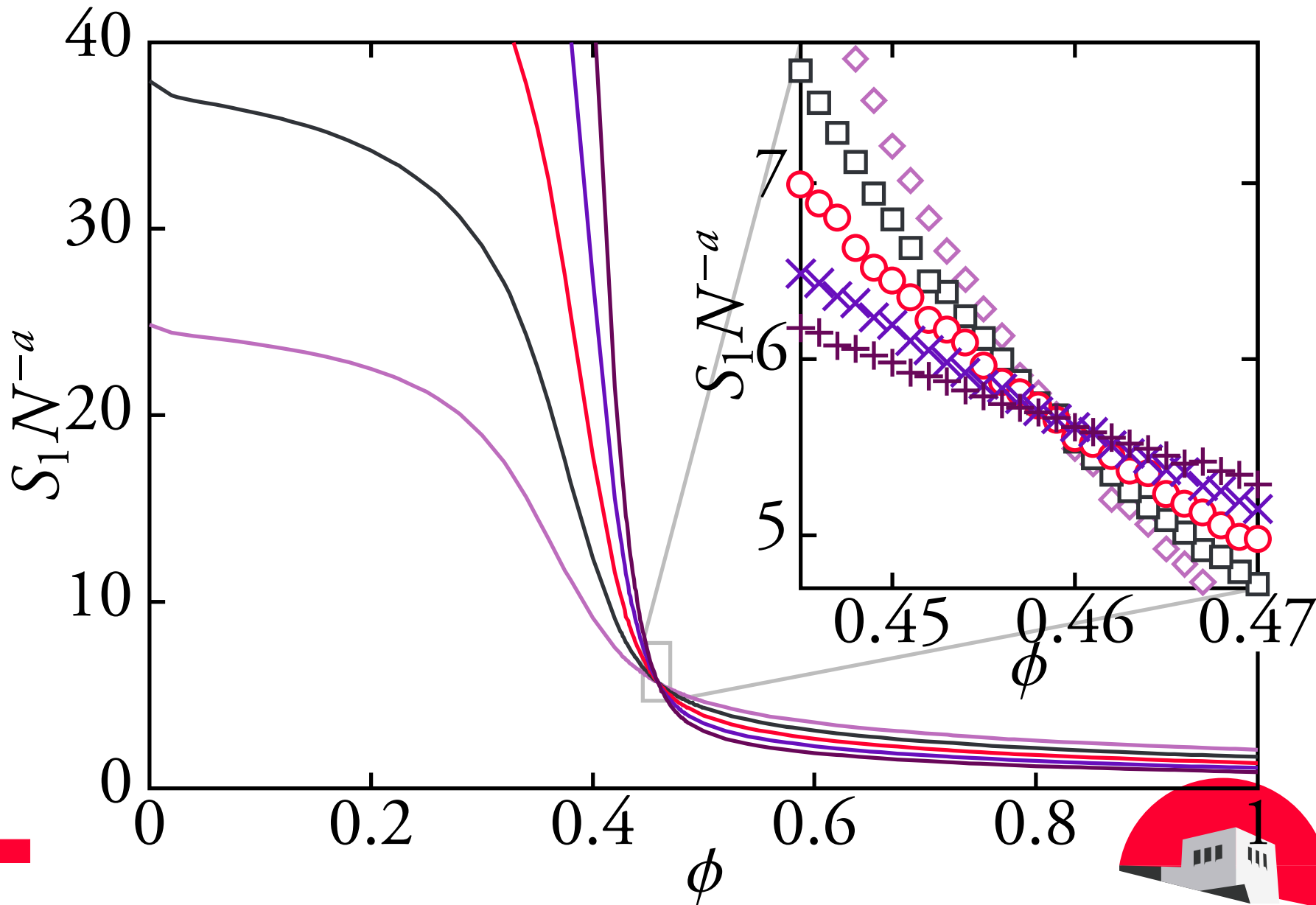


Assume a critical scaling form:

$$S = N^{-a} F\left(N^b(\phi - \phi_c)\right)$$

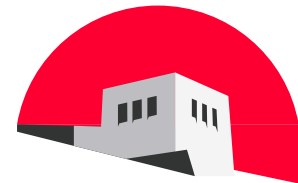
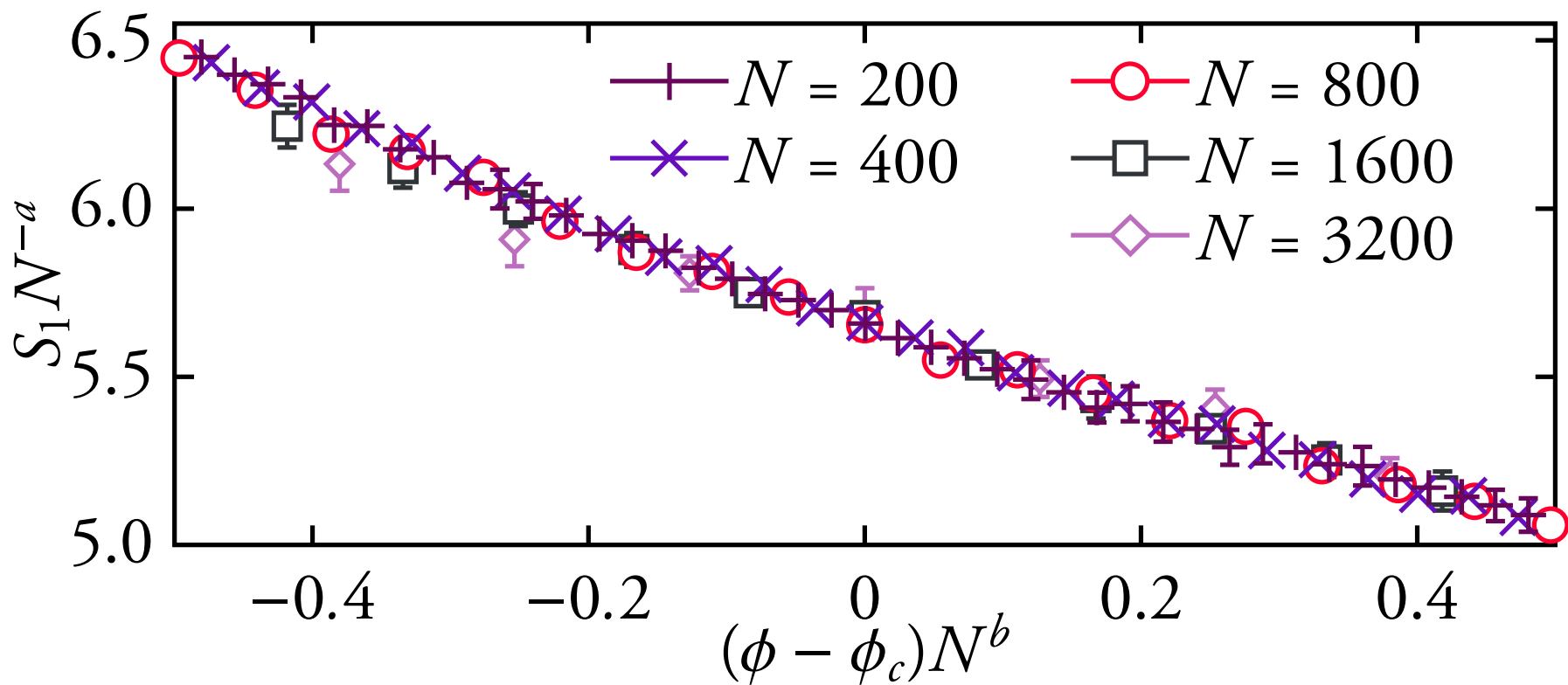


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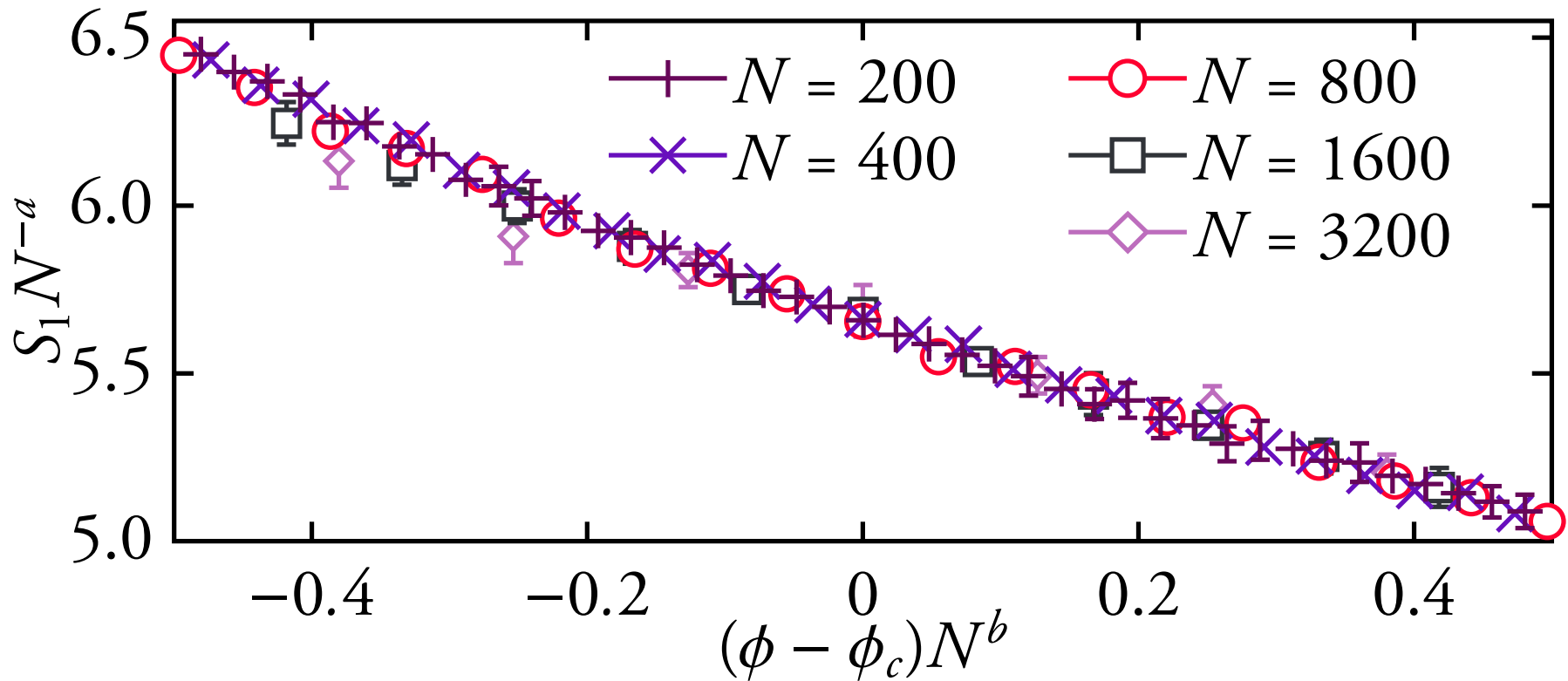




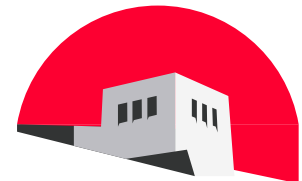
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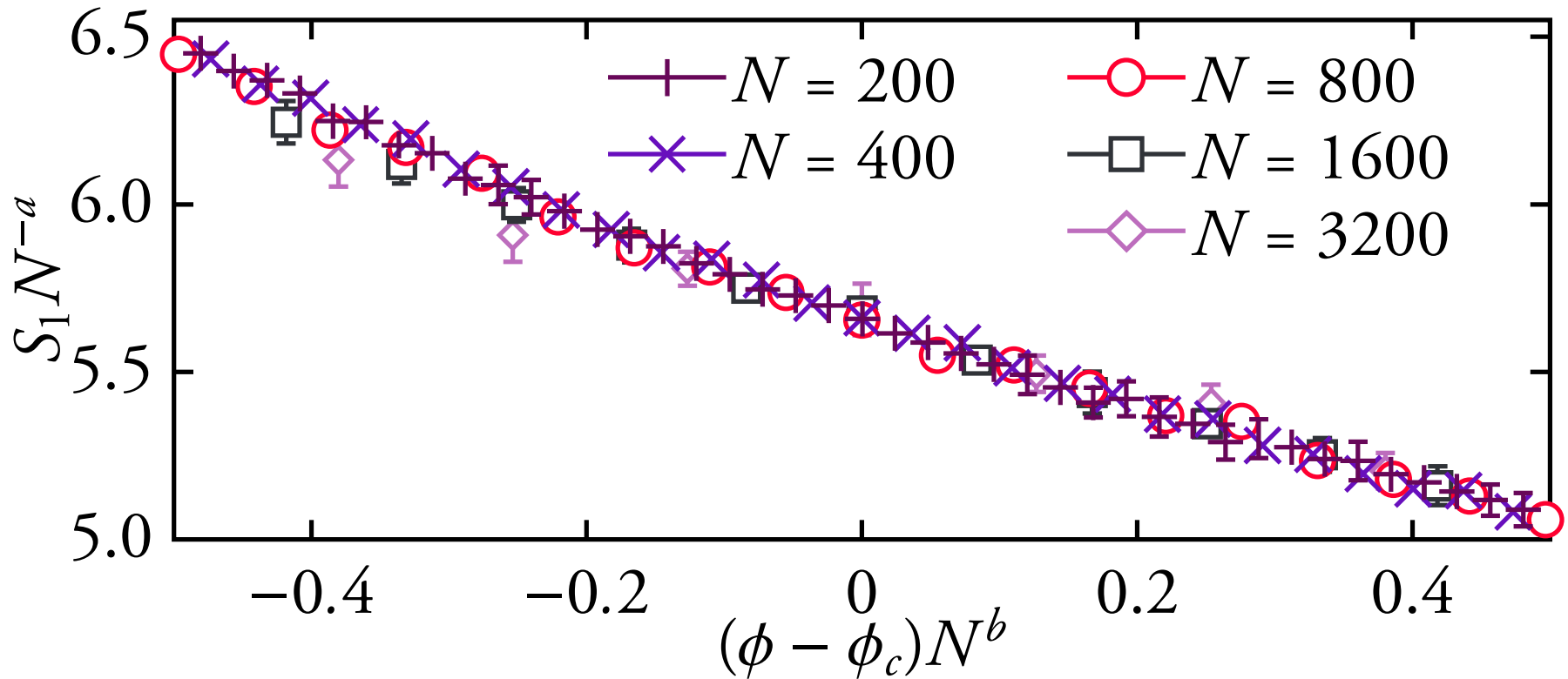
# finding the phase transition



$$a = 0.61 \pm 0.05, \phi_c = 0.458 \pm 0.008, b = 0.7 \pm 0.1$$



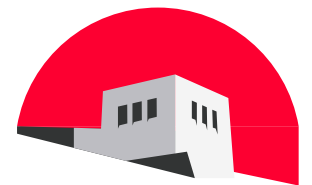
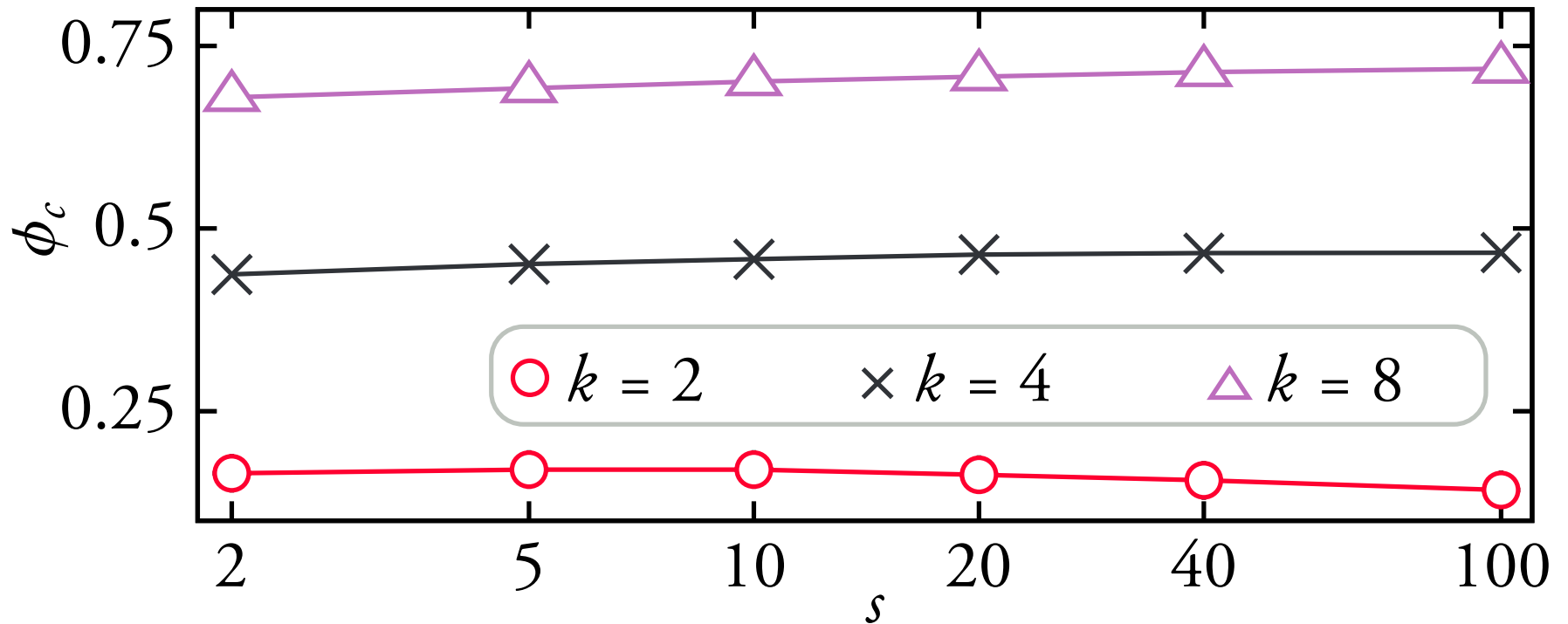
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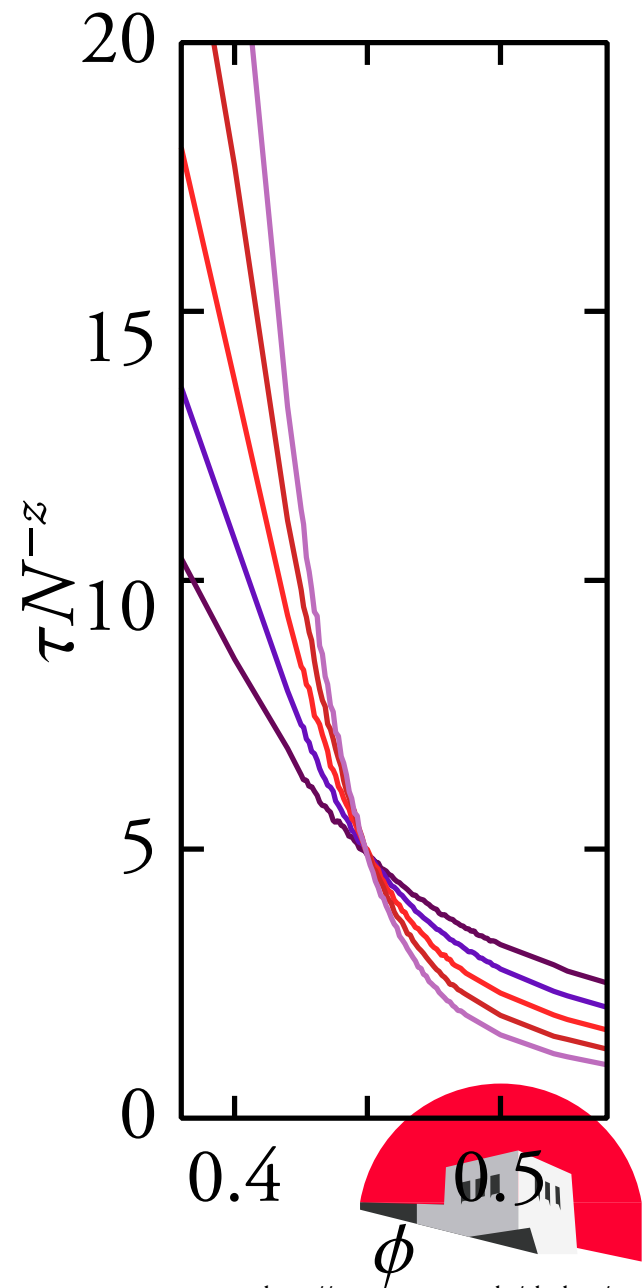
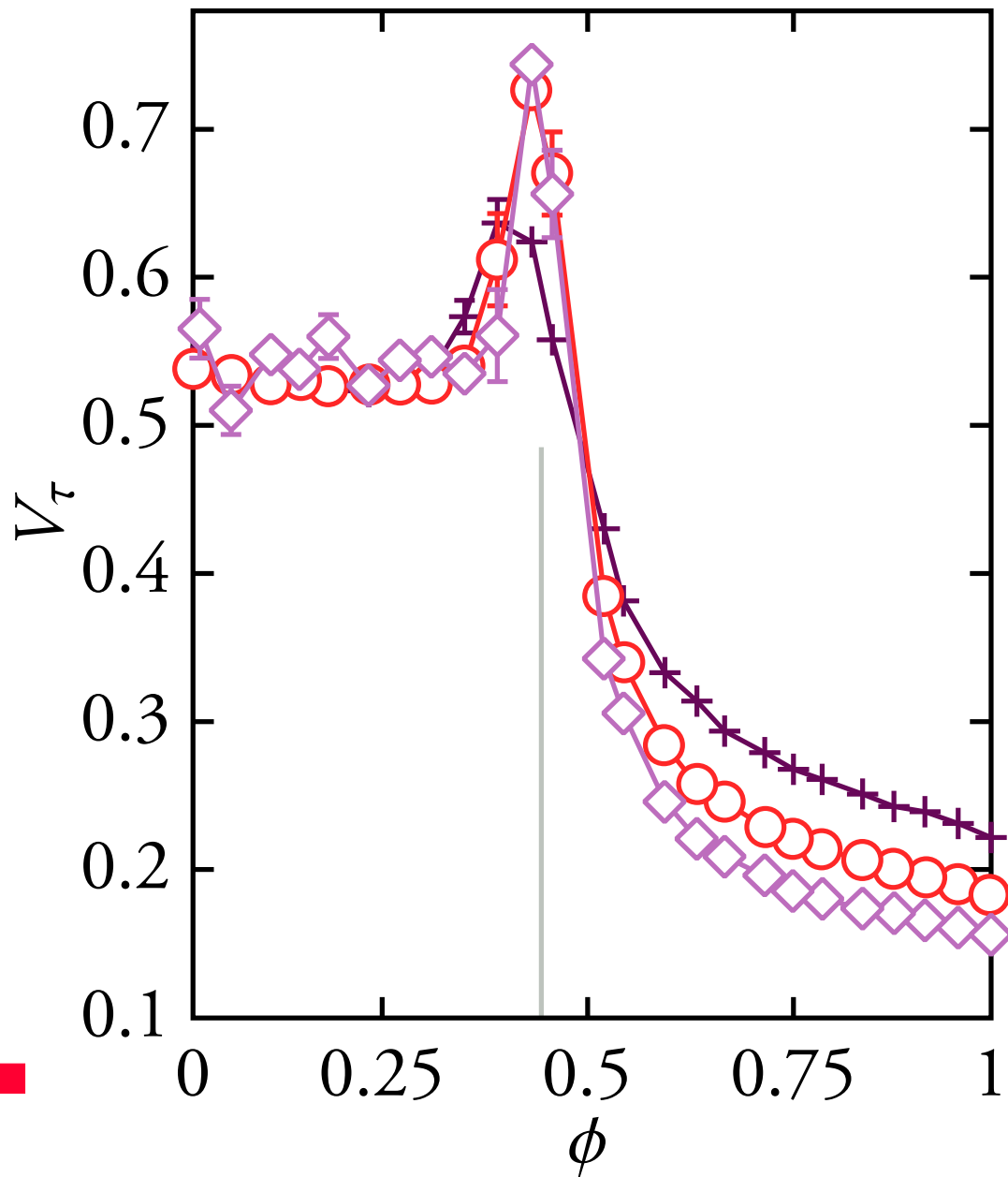
$a = 0.61 \pm 0.05$ ,  $\phi_c = 0.458 \pm 0.008$ ,  $b = 0.7 \pm 0.1$   
random graph percolation:  $a = b = 1/3$



# phase diagram



# dynamic critical behavior

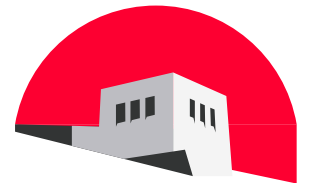


# conclusions

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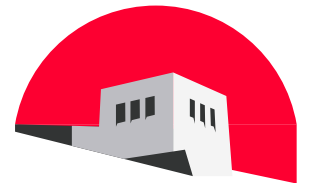
- ★ We have proposed a simple, non-equilibrium model for the coevolution of networks and opinions.



# conclusions



- ★ We have proposed a simple, non-equilibrium model for the coevolution of networks and opinions.
- ★ The model undergoes a second order phase transition between: One state of clusters of similar sizes. One state with one dominant cluster.



# conclusions



- ★ We have proposed a simple, non-equilibrium model for the coevolution of networks and opinions.
- ★ The model undergoes a second order phase transition between: One state of clusters of similar sizes. One state with one dominant cluster.
- ★ The universality class is not the same as random graph percolation.

